DOCUMENT RESUME

ED 022 688 SE 004 947

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SCHOOL MATHEMATICS STUDY GROUP REPORT NO. 1, THE PROGRAMED LEARNING PROJECT.
Stanford Univ., Calif. School Mathematics Study Group.
Spons Agency-National Science Foundation, Washington, D.C.
Pub Date 66
Note-96p.
EDRS Price MF-$0.50 HC-$3.92
Descriptors-*ALGEBRA, CURRICULUM, *CURRICULUM DEVELOPMENT, CURRICULUM EVALUATION, GRADE 9, INSTRUCTION, *MATHEMATICS, *PROGRAMED INSTRUCTION, *SECONDARY SCHOOL MATHEMATICS
Identifiers-National Science Foundation, School Mathematics Study Group

This manual is written for programers whose specific assignment is the preparation of programed texts in mathematics based on the work of the School Mathematics Study Group (SMSG). Detailed information regarding the attempt to program the SMSG ninth grade course, "First Course in Algebra," is provided. Reported are (1) information on "The Manual for Programmers," which was drafted for use in a summer workshop and by writing teams, (2) the workshop which was arranged to acquaint a core of writers with programed instructional materials and the preparation of sample units, and (3) the mode of operation of each of the twelve writing centers consisting of mathematicians and/or mathematics teachers. The result of this project was the production of the SMSG "Programed First Course in Algebra" which was field tested in the 1962-63 academic year in several schools throughout the country. (RP)

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SMSG REPORTS

No. 1

The Programed Learning Project

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Financial support for the School Mathematics Study Group has been provided by the National Science Foundation.

A preliminary draft of this report was prepared by Mr. Leander Smith in the spring of 1964. Mr. Smith had been the project coordinator for the SMSG Programed Learning Project since its inception in the spring of 1961.

The statistical analyses contained in the report were carried out by Mr. Ronald Pyszka, Research Assistant at SMSG headquarters, during the 1964-65 academic year. Mr. Pyszka prepared a second draft of the report which incorporated the results of the statistical analysis.

The final draft of this report was prepared by Mr. William Chinn, also a member of the SMSG headquarters staff, as well as a participant in the writing team which prepared the programed text. This final draft, prepared in the fall of 1965, incorporated comments and suggestions made by members of the SMSG Programed Learning Panel in response to the second draft of the report.

TABLE OF CONTENTS

The SMSG Programed Learning Project

Page

Recommendations for the Study 1
The SMSG Panel on Programed Learning 2
The Manual for Programers 3
The Workshop 3
The Writing Centers 5
Experimental Centers 7
The Summer Writing Session 7
Test Teaching the Preliminary Editions 10
Teacher and Student Reactions 13
The Hybrid 15
PLP Centers 17
Follow-up Study of Retention 30

Appendices

A Test Score Estimates 32
B Lo-Hi Cognitive Estimates 35
C Estimates of Retention 38
D Levels of Intellectual Activity 39
E Sample Summary Guide for Writing 40
F Manual for Programers 43

THE SMSG PROGRAMED LEARNING PROJECT

Recommendations for the Study

Following B. F. Skinner's articles, "The Science of Learning and the Art of Teaching"[1] in 1954 and "Teaching Machines"[2] in 1958, there appeared many programed texts with subject matter ranging from statistics to foreign language training. Extensive programing of mathematics by persons expert in programing but not in mathematics has led the SMSG Advisory Board to decide that a careful study of programed learning should be undertaken with specific reference to SMSG materials. It was felt that although programed instruction had evolved as a psychologist's tool, the SMSG efforts should be carried out jointly by mathematicians, teachers of mathematics, and psychologists under the supervision of a Panel on Programed Learning.

In 1961, an ad hoc committee on programed learning of mathematics was appointed to advise the Director of SMSG on procedures for carrying out the decision of the Advisory Board for a careful study. The committee met at Harvard University on May 2, 1961. It was agreed by this committee that, although the SMSG 7th grade course would be technically easier to program, an attempt to program the SMSG 9th grade course would provide more useful information. The following recommendations were made:

1. Mathematicians from both high school and university levels should be taught to construct programs.

2. The Director of SMSG should locate a suitable number of qualified individuals who could devote part of their time during the 1961-62 academic year to the project.

3. Each section of the SMSG First Course in Algebra should be translated into programed form.

4. A variety of programs should be prepared.

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[1] Lumsdaine, A. A., R. Glaser, Teaching Machines and Programed Learning: A Source Book, DAUI of NEA, Washington D.C., 1960, pp. 94-113.

[2] _________, op. cit., pp. 137-72.

5. The program writers should retain the content and sequence of the SMSG First Course in Algebra in order to make comparisons possible.

6. The problem of motivation should be kept in mind in writing the program.

7. Feedback should be obtained for the various drafts from students who try the program.

8. The objective of the first part of this SMSG study should be to verify the hypothesis: that the SMSG material can be presented as effectively according to prescribed criteria, through programs as through conventional classroom-textbook procedures.

9. Evaluation of any program should involve the use of tests which require students to solve problems and recognize concepts outside the text.

The translation of the SMSG First Course in Algebra was to be accomplished by a small group consisting of a university mathematician and classroom teachers thoroughly familiar with the course. The variety of programs should include a constructed response program (featuring small steps at low error rate with some branching and occasional inclusion of large steps as variations to be experimented with), a multiple-choice program (featuring a scrambled book format with explanations of incorrect answers using the size of the unit as a variable to be experimented with), and a section (sections) to be used as supplement(s) to a textbook. With reference to the problem of motivation, the program writers were to take a close look at what the classroom teachers do to supplement the motivation supplied by the text. For feedback on the various drafts, it was recommended that the first draft be used to obtain detailed comments and information from a small number of students, involving the possible use of machines.

The SMSG Panel on Programed Learning (June 1961)

Following the recommendations of the preliminary committee, the Director appointed a Panel on Programed Learning consisting of R. C. Buck, University of Wisconsin; E. E. Hammond, Jr., Philips Andover Academy; L. D. Hawkinson, San Francisco Unified School District; J. G. Holland, Harvard University, W. J. McKeachie, University of Michigan; E. E. Moise, Harvard University; H. O. Pollak, Bell Telephone Laboratories; and D. W. Taylor, Yale University.

The first meeting of the SMSG Panel on Programed Learning was held at Harvard University on June 16, 1961. It was agreed that a committee consisting of V. H. Haag, H. O. Pollak, and C. E. Rickart (members of the writing team on the SMSG First Course in Algebra), should prepare a chapter summary and statement of objectives for the algebra.

Other specific operational recommendations of the Panel include the following:

1. Program items, written on cards with answers on the back should be tried on a small number of students to detect gross errors, ambiguities, etc., and revised on the basis of student feedback.

2. The program should be reviewed by mathematicians, teachers of mathematics, and psychologists for both quality of content and quality of programing. Then, a revision should be prepared.

3. Additional motivational material may be added if the same material is made available to classes using the conventional text during the experimentation in 1962-63.

The Manual for Programers (July 1961)

Various stages for the implementation of the project include: the production of programed materials, trial runs with groups of students, evaluation of results, revisions, and further study.

Preliminary to the full scale production of materials, a Manual for Programers was drafted by the Coordinator of the project for use in a summer workshop and by the writing teams. The manual was intended as a basic document to provide persons having a thorough knowledge of the mathematics with information that they would need for the actual construction of programs. It included background material on the psychology as described by B. F. Skinner, an overview of the state of the art of programing, specific recommendations for the analysis of the content of the SMSG First Course in Algebra, and notes on the construction of items.

After the draft manual was checked by members of the Panel for glaring errors in content, it was then used by the Workshop in August 1961 and revised for use by the writing teams during 1961-62. As more knowledge was gained about programing, materials were added to or deleted from the Manual (see APPENDIX F for revised manual).

The Workshop (August 1961)

During the summer of 1961, the Director of SMSG and the Project Coordinator located sufficient manpower to establish twelve writing centers to work on a part-time basis during the 1961-62 academic year. From each of these centers, representatives were assembled in New Haven, where the 1961 SMSG Summer Writing Session was being held, for a workshop from August 7 to 18. The following people attended the Workshop: D. Blakeslee, San Francisco; M. P. Bridgess, Boston; F. Jacobson, New Haven; H. Jones, Stillwater; W. Matson, Portland, Oregon; O. Peterson, Emporia, Kansas; P. Redgrave, Norwich Connecticut; W. Storer, Des Moines; and H. Swain, Winnetka. In addition to the participants from writing centers, SMSG secured J. C. Hammock, a psychologist at Bell Telephone Laboratories and P. I. Jacobs, a psychologist from Educational Testing Service.

The purposes of the Workshop were:

1. to acquaint a core of writers with programed instructional materials and the procedures for producing them;

2. to prepare sample units in constructed response and multiple-choice formats which could serve as models for the center writing teams;

3. to take on writing assignments and suggestions for the organization of the twelve centers.

To accomplish the first purpose, the draft of the Manual for Programers was examined, criticized, and used as the basis for writing sample units. In preparing sample units, Sections 1-1 and 6-5 of the SMSG First Course in Algebra were selected for the programing in the Workshop. Participants formed two writing teams; one to produce a constructed response program, the other to produce a multiple-choice program. With the experience of writing as a team in New Haven, the participants were then able to anticipate problems of producing programs and orienting other members of the centers. Lumsdaine and Glaser's Source Book and Keller's Reinforcement Theory[1] provided psychological background on programing; literature produced by the Center for Programed Instruction (N.Y.C.), Doubleday Tutor-Texts, TEMAC, and Holland and Skinner's Analysis of Behavior[2] provided examples of existing programs.

It is important to note that in this phase of the Project, an attempt was made to produce two "pure form" programs. Each program was to consist of either all constructed response or all multiple-choice items. The order of topics and the point of view of the SMSG *First Course in Algebra* were to be preserved.

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[1] Keller, F. S., Learning: Reinforcement Theory, Random House, New York, 1954.

[2] Holland, J. G., B. F. Skinner, Analysis of Behavior, McGraw-Hill Inc., New York, 1961.

The Writing Centers (Academic Year 1961-62)

Each of the twelve writing centers consisted of three or four mathematicians and/or mathematics teachers whose acquaintance with SMSG *First Course in Algebra* stems from participation in the original writing team, the evaluation centers, or teaching SMSG mathematics to teachers. With knowledge of the mathematics well in hand, it was felt that these people could be provided with the information that they would need for the construction of programs. The centers were located in Boston, Chicago, Des Moines, Emporia (Kansas), New Haven, Palo Alto, Portland (Oregon), San Francisco, and Stillwater (Oklahoma). Three centers were located in Chicago, two in Palo Alto, and one in each of the other cities named. The sections of the *First Course* were partitioned among the centers for programing.

Twelve critics were selected to review the writing done in the centers; six to review the constructed response materials and six to review the multiple-choice materials. Each reviewer was to comment specifically on the frames and to react generally to the section, using the following guide:

Check on the scale where you feel this unit ranks.

1.	Unit length:	0	1	2	3	4	5	6
		too long			all right			too short
2.	Unit continuity:	0	1	2	3	4	5	6
		choppy			smooth			forced
3.	Unit difficulty:	0	1	2	3	4	5	6
		too hard			all right			too easy
4.	Unit vocabulary	0	1	2	3	4	5	6
		too hard			all right			too easy
5.	Frame length:	0	1	2	3	4	5	6
		too long			all right			too short
6.	Unit boredom:	0	1	2	3	4	5	6
		low			all right			high
7.	Unit Panel Use:	0	1	2	3	4	5	6
		poor			all right			excellent
8.	Reinforcement:	0	1	2	3	4	5	6
		poor			all right			excellent

9. Unit per cent (C)	0	10	20	30	40	50	60
est. error: (S)	0	5	10	15	20	25	30

Other comments on the unit or subunit ______________________________

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Mode of Operation

To show clearly the operation of the writing centers during the academic year 1961-62, it is best to take a representative center and discuss its mode of operation. Each center consisted of one mathematician and three teachers of mathematics operating on a part-time (2 days per month) basis. The mathematician served in each case as a consultant on the mathematics content and as the final editor of the material. Each member of the writing team first generated an outline of the sections to which he was assigned. This outline was based on the content and organization of the SMSG First Course in Algebra and the Summary and Objectives outlined by Haag, Rickart, and Pollak (See APPENDIX E and APPENDIX 4 in Manual p. 91). The problem set of each section was surveyed for problems which occurred early in the textual material. A first draft was then prepared on file cards.

In the case of the constructed response program, the correct response was placed on the back of the card; in the case of the multiple-choice program, the alternatives were written on subsequent cards (usually of different color for ease in identifying). The first draft was then passed to other members of the center and to the center chairman for suggestions. On the basis of these comments the original writer prepared a second draft version. Next, the revised draft passed to the center chairman for editing, who, in turn sent the draft to SMSG (then at Stanford University) for ditto reproduction.

Each reviewer then commented on the particular phase of the work in which he was best fitted: the psychologist commented on the programing per se, the mathematician on the correctness of the presentation of the mathematics contained in the First Course, and the teachers on the appropriateness of language to the high school student. All the reviewers' comments were returned to Stanford where they were consolidated by a typist and sent back to the writing team as guidelines for further revision.

On receipt of the comments from the reviewers, the writing team reconsidered the language, mathematics, and programing technique, and prepared another draft which was forwarded to SMSG for lithographing.

Experimental Centers (Spring 1962)

Once the lithographed version of the first eight chapters of the Programed First Course in Algebra was produced (January 1962), classes were sought to try out these materials (to start in March 1962). It was suggested that eighth grade students, ready to begin algebra would be appropriate students. Other classes were to include: slow ninth grade students, students in upper grades who had not taken algebra, or repeating students whose earlier experience was not with SMSG-like mathematics.

A two part pre-test was designed to serve as an inventory of skills and concepts already in the command of the students. The first part surveyed material covered in the 7th and 8th grade SMSG classes; the second part was to be used as both pre and post tests.

In this pilot study, data collection and summarization was not adequate. Teachers' summaries of student results were sketchy, at best, and student response sheets themselves were too voluminous to enable summarization in time for the 1962 Summer Writing Session. The pre and post tests, however, were useful in the generation of revised testing (PLP-2 and PLP-1). Teachers indicated too, that chapter tests would also be a valuable adjunct to the program.

The Summer Writing Session (Summer 1962)

The 1962 Summer Writing Session participants were selected on the basis of demonstrated ability in writing programed material and in meeting deadlines; role determination based on the need for a psychologist, mathematicians, and teachers; perception of the problem and goals of SMSG; and freedom to devote eight weeks of their time to this project. The participants were: John D. Baum, David W. Blakeslee, M. Philbrick Bridgess, Mrs. Marjorie French, James E. Gilbert, Arthur A. Hiatt, Stephen Hoffman, Mrs. Margaret Matchett, William W. Matson, Mrs. Persis O. Redgrave, Winfield Roberson, Robert E. Schweiker, William Storer, George Truscott, and Mrs. Helen Wehe. The participants were subdivided into two writing teams: one to revise the constructed response program, and the other to revise the multiple-choice program. Mr. Bridgess chaired the multiple-choice (Form MC) team; Mr. Blakeslee chaired the constructed response (Form CR) team.

It had become clear during the course of the year when the writing was done at the twelve individual centers (1961-62), that the ability of each writer to reproduce the material of the course in a language for 9th grade students to read was widely diverse. It had also become clear, as the writing progressed, that the idiosyncratic behavior of the individual writers was to appear in the Programed First Course in Algebra as writers sought to elaborate on material treated briefly in the original text, to create proofs where none had appeared, and to replace problems with their own versions of "good items to ask". In the summer writing sessions where two editors passed judgment on the content, there was more uniformity in format, and the divergence from the original text was reduced--in keeping with the stated objectives of this study.

Since the student response sheets from the spring try-out had not been processed by the time the writing teams assembled on June 22, 1962, the writers set aside student try-out materials and worked from the lithographed version as a first draft. Although the twelve widely separated centers had spent most of their time on the first eight chapters and had left Chapters 9-17 in rough ditto form, the 1961-62 efforts still represented a real starting point; their efforts enabled the summer writers to generate materials for use in schools in the 1962-63 academic year.

With each of the two teams working during the Summer Session, considerable time was spent at the beginning of the session on style, format, semantics, and the more basic problem of screening the materials produced by the center writing teams. The mathematicians on the summer writing team began by preparing a first version of the lithographed version of the course. In some cases, this necessitated re-examination of the First Course in Algebra and re-evaluation of the goals of that course. Some of the areas that presented challenge were in the programing of problem sets, proofs, and review topics. In addition, each team encountered problems that might be considered characteristic of that particular format.

The writers of the multiple-choice program spent considerable time anticipating distractors (alternatives which require careful discrimination and knowledge of skills and/or concepts). The multiple-choice format enabled some preservation of text but suffered in its ability to elicit computational practice because of the form of presentation. The multiple-choice program that was produced that summer, although a scrambled book in form, was essentially a single-track program through which a student would proceed. Very little effort was made to capitalize on the multiple-track program that might be possible in later editions, given more time to write.

The writers of the constructed response program, on the other hand, found the fragmentation of the earlier version quite a problem. There was an initial tendency to use lengthy sets of problems for practice, overcuing, and little synthetic material. Finding a need for synthesis, the writers redefined "frame". A "frame" had been conceived as a short passage, three to five lines, with a significant word or words missing. The response sought was then to contribute to the learning of the skill or concept. Each frame was then defined as a "bit" or "response eliciting item".

As the summer progressed, the fragmentation of content into lengthy sequences of "bits" began to draw reaction from the writers. They began to group exercises of similar nature into a box or frame; they then sought to use the "frame" as a means of identifying a recognizable step in the learning process. Each frame evolved as a conceptual unit; that is, an identifiable step toward the mastery of a skill or concept. By having the manuscript passed among each of the writers, the changes in format were quickly assimilated and further suggestions resulted.

The ease of programing seemed to depend on the explicit structure of the content presented in the text and the independence of topics being developed. It was relatively easy to program Section 1-1 (Sets and Subsets) where the content prescribes an order of presentation. On the other hand, it was very difficult to program Section 1-2 (Sets and the Number Line) where the content of the original text did not suggest an orderly presentation and the number of topics which might be included had to be considered. Chapter 7 (Properties of Multiplication) and Chapter 12 (Polynomial and Rational Expressions) were most difficult to program. As mentioned before, programing of proofs was a challenge. To overcome part of the difficulty in programing proofs, use was made of the constructed panel in the CR format. Here, the student was led to complete steps (or to supply reasons for steps) in a proof, transferring these responses to appropriate blanks in a Panel to complete the proof. The constructed proof was then compared with a model proof as it might appear. Thus, the Panel construction served to integrate the small pieces of the proof into a whole. Also, in the process of reconstruction, more than one pass was made in the proof.

The results of the 1962 Summer Writing Session were three-fold:

1. The SMSG Programed First Course in Algebra, Form CR: a constructed response programed text with 13,552 responses in a total of 1,712 pages. All

items were written on right hand pages only to allow a right hand slider to be used to conceal confirmations; then, having gone through the book using right hand pages, the student flips the book over and again used right hand pages (the left hand pages when the book is in normal position). Form CR was "perfect bound" in six volumes. One of the problems caused by the 180° rotation of the book had to do with reference to various Panels which had been printed in the back sections of the book when it is normal position. After the flipping, the Panels appeared in an upside down position.

2. The SMSG Programed First Course in Algebra, Form MC: a multiple-choice response programed text with about 2,200 items in a total of 2,357 pages bound in six volumes. The books were scrambled within sections; that is, each section began in position 1A and was paged independently from the rest of the text. A student's path through a section might thus read: 1A, 10C, 4A, 9B, 1B, 11A, 8A, 6C, 2D, 13D, 5B, 11B, 7C, 10B, 12B, 3D (Section 1-1, Form MC), according to the response he gives to each item.

3. The PLP Chapter Tests: a series of tests covering the content of the 17 chapters of the First Course in Algebra; also, pre and post tests based on the preliminary versions written during the 1961-62 academic year. The pre and post tests were designed to cover the following material:

PLP-1 -- inventory of 7th and 8th grade mathematics;

PLP-2 -- first semester algebra;

PLP-3 -- second semester algebra with overlap.

Test Teaching the Preliminary Editions (1962-63)

In the spring of 1962, a design was planned for experimentation during the 1962-63 academic year. Schools throughout the country were asked if they would be interested in test-teaching the SMSG materials. These schools were selected on the basis of earlier participation in SMSG test-teaching of the First Course in Algebra and on the recommendations of members of the writing team.

The design called for testing the following four modes of instruction:

MC - the multiple-choice programed text;
CR - the constructed response programed text;
MCR - the constructed response machine format (unbound CR in Koncept-O-Graph KOG-7);
9F - standard SMSG First Course in Algebra.

Each mode was to be used under certain of the following conditions:

Time-paced with a teacher:

> assigned with a due time or due date in blocks or sections with a teacher to control disbursement and to answer questions and discuss or explain any mathematics.

Time-paced with a monitor:

> assigned with a due time or due date in blocks or sections with a monitor to control disbursement and to answer nonmathematical questions.

Self-paced with a teacher:

> in classroom or home with a teacher to record time and to answer questions and discuss or explain any mathematics.

Self-paced with a monitor:

> in classroom or library with a monitor to record time and to answer nonmathematical questions.

Students were to be regular ninth grade algebra students, accelerated eighth grade students, or remedial tenth grade students. Teachers were asked to designate the ability level of their classes, but no standardized testing was administered for this purpose. All students were given pre-tests (PLP-1 and PLP-2), the seventeen chapter tests, and post-tests (PLP-2 and PLP-3). All student test answer sheets were scored and returned to SMSG for analysis.

The response of the schools was generally very good. As it turned out, however, teachers used the statement of the project coordinator, "If for some reason, students fail to benefit from the program, SMSG should be contacted and action taken to save the students." as an escape, a method of reverting to "normal" classroom instruction. Often, the notion of self-paced classes was so aversive to teachers that they established "minimum" rates which, in turn, became expectancy levels. Almost all self-paced classes reverted to some form of time-pacing at some point during the year. Since they thus constituted a conglomeration of methods of presentation rather than pure form of self-pacing, they were eliminated from the analysis.

The four treatment groups chosen for analysis were MC (multiple-choice), CR (constructed response), MCR (constructed response, machine format), and 9F (standard SMSG algebra test)--all time-paced at 9th grade. The number of students in each mode is given in TABLE NUMBER 1 .

TABLE NUMBER 1

Mode	N
CR	260
MC	341
MCR	142
9F	138

The MCR classes were supervised by monitors while all others were supervised by teachers. It is believed, though, that there is no difference between the way the monitors handled their classes and the way the teachers handled theirs.

Since the four treatment groups differed on PLP-1 (pre) and PLP-2 (pre) (measures of initial level of achievement), a covariance technique was used to give estimates of what the scores on PLP-2 (post) and PLP-3 (post) would have been had the four treatment groups been comparable on the two pre-tests. These estimates, or adjusted means, were tested for significance of difference among treatment groups. (See APPENDIX A.)

The analysis was done twice, once with PLP-2 (post) as the criterion variable (adjusting for differences on PLP-1 (pre) and PLP-2 (pre)) and once with PLP-3 (post) as the criterion variable (adjusting for differences on PLP-1 (pre) and PLP-2 (pre)). With PLP-2 (post) as the criterion variable, the analysis indicated that both the Constructed Response and the Multiple-Choice modes are significantly better than 9F while there is no significant difference between them (all significance is at least the .05 level). Also, both CR and MC are significantly better than MCR while there is no significant difference between MCR and 9F. With PLP-3 (post) as the criterion variable, the results were identical except that the difference between CR and 9F was not significant.

In interpreting the results of those comparisons involving MCR, it is necessary to recall that the classes using this mode were administered by monitors while the classes using the other modes were administered by teachers. It is impossible to say what role this teacher-monitor variable plays in the observed differences between MCR and the remaining three modes.

In TABLE NUMBER 2 the adjusted means for PLP-2 (post) and PLP-3 (post) are presented. In addition, the mean differences between PLP-2 (pre) and PLP-2 (post), adjusted for differences on PLP-1 (pre), are presented. It is evident that, while many of the differences among treatment groups are significant, they are small in amount.

TABLE NUMBER 2

Mode	Adjusted Means		
	PLP-2 (diff)	PLP-2 (post)	PLP-3 (post)
CR	9.60	16.66	10.72
MC	9.16	16.22	10.93
MCR	6.23	13.86	9.28
9F	7.48	14.90	9.89

Teacher and Student Reactions (1962-63)

There was intense negative reaction from some of the teachers and students against the boredom they found resulting from the programs. Much of this reaction appeared to be generated by the sameness of format. In the case of the constructed response program, boredom was intensified among the faster students who were forced to go through long series of items designed primarily to teach the slower students. In the multiple-choice program, which provides for branching and skipping to care for the individual differences, there was objection to the "page-flipping" involved in the scrambled book technique; they also objected to being unable to review systematically without retracing all the steps in the program. In both cases, frustration was heightened by the books falling apart.

In February 1963, a questionnaire was sent to all 150 teachers in the Programed Learning Project to gain insights on what future action SMSG should take in the area of programed learning. From these, 141 responded with the following sorts of information:

Number of years experience teaching SMSG Algebra:
(0 years - 35 teachers, 1 - 42, 2 - 21, 3 - 23, 4 - 11)

Proportionately, the greatest support for programed learning came from those teachers with four years of experience with the algebra: 8/11 as opposed to the overall total of 73/132. Of the 140 teachers reporting, 79 favored programed learning, 61 were opposed. Teachers of the eighth, eleventh and twelfth graders favored the program most; in fact, teachers in 15 of the 18 eighth grade high-ability groups would use the material again. Favorable reactions were reported in 34 of the 63 schools using Form CR and in 28 of the 42 schools using Form MC. The machine users were negative in a ratio of 2 to 3.

Some specific comments by a number of teachers of Form CR are listed below. Relating to: writing style--"Text is too wordy", "The program appears too easy", "The students feel lost in too many frames"; form of presentation--"Most students tend to move too slowly when allowed to set their own pace", "Self-paced instruction does not seem appropriate for eighth grade students"; personal reaction--"At the moment, I feel so negative about the material that I could not be objective"; observation of student response--"...most students are happy to know exactly and instantly when they are wrong", "It is too easy to cheat." content--"There appears a lack of overview of subject matter"; accompanying tests--"The tests are too hard", "The tendency to do work mentally handicaps them when they get to tests where the steps are not outlined for them"; overall weakness--"Most of the weaknesses of the programed text are the weaknesses of the original text"; and suggested modifications--"Frames should be grouped by single ideas", "I suggest some flexibility to allow teachers to choose certain additional or fewer exercises dictated by student need", "More problems need to be given after the material is presented step by step", "The program needs an index", "Answer sheet could be arranged better to facilitate grading".

Some criticisms from teachers using Form MC are listed below. Relating to: student response--"Students admit they are learning but feel it is more work than would be necessary in a regular class. They find it difficult to review for a test", "The better students seem to like the MC format better than classes using the CR", "Students are bored; they need more problem-solving"; presentation--"Program was exciting initially. It needs variety and possibly additional problem sets", "Good readers seem to do well; poor readers just can't seem to get anywhere", "Program is boring. The chapters would be better partially programed so the teacher could teach and use the program as homework"; shaping of learning--"There is no place where the student learns to put a problem in good form and arrange his work in an efficient manner".

From teachers who used teaching machines (Konceptograph, KOG-7) the reaction was almost universally:

"The machines are not satisfactory at all--about half are now (January 1963) unusable because of mechanical deficiencies." As a result, SMSG recalled the machines and destroyed all but a few which were retained for experimentation on controlled testing. The program used in the machines was Form CR in an unbound format to enable single sheet-fed operation.

The most often expressed comments on the questionnaire about the program dealt with the presentation and format. Both students and teachers missed classroom discussions, without which they were unsure that progress was being made. The students wanted to hear the material summarized and "pulled together". The students liked to know when they were wrong and why the error had been made. On the matter of format, they claimed the presentation was not sufficiently varied.

The Hybrid (Spring - Summer 1963)

From the widespread dissatisfaction of teachers and students in the 1962-63 classes came the feeling that the "pure form" programs had made SMSG mathematics less palatable than the text on which they were based. Hence, in the spring of 1963 the Project Coordinator prepared a sample which combined the features of both earlier versions into a hybrid program. The features which were felt essential included:

(1) immediate confirmation on most constructed response items;

(2) confirmation and correction of errors on multiple-choice items without scrambling the text;

(3) inclusion of conventional textual material;

(4) inclusion of reviews and problem sets with confirmations given in the back of the book;

(5) inclusion of optional sections for more able students and for students who want or need additional help;

(6) inclusion of index, table of contents, tailor-made response sheets;

(7) variation in format to relieve boredom;

(8) provision for skipping items by those who have successfully completed criterion items.

With the sample hybrid format and experienced writers of constructed response and multiple-choice programs, the 1963 SMSG Summer Writing Session began. Mrs. Persis Redgrave was asked to chair and edit the work of the writing team consisting of D. Blakeslee, M. P. Bridgess, F. Elder, J. C. Hammock, F. Jacobson, M. Matchett, W. Matson, L. W. Smith, W. Storer, M. Wecheler, and M. Zelinka.

During late spring 1963, three classes were chosen from classes using each of the three texts (9F, MC, and CR). The students in these classes were ninth graders whose mean scores on PLP-1 were approximately equal: CR, 11.47; MC, 10.93; 9F, 11.11; total, 11.17. Item analysis of the chapter tests for these students was then done to provide a guide for the 1963 Summer Writing Team. In addition, one in every eight students using mode CR and mode MC in the 1962-63 testing was required to return his response sheets. For students using mode CR, a summary of the number of errors made on each item in the program, together with the total number of attempts, was prepared.

Since existing programs in mathematics had tended to be reproductions of the work of Skinner (CR) and Crowder (MC), there were no printed models of a hybrid for the writing team to begin with. Hence, as soon as Section 1-1 had been written, "sample pages" were set up by the typists and printed to show what the text would look like when finished. A fine gray screen was laid over the confirmation areas and all items requiring a response were framed. The gray zone identified material to be red after students had made their responses to specific items. Tailor-made response sheets were prepared, to be printed, bound into the back of each volume, perforated for students to remove.

After a few sections of Form H (the hybrid format) had been written, a student was found to go through the program. Having studied no algebra previously, he worked under the surveillance of Dr. Hammock who recorded the time spent, checked over the errors (many of them typographical) and administered the testing. In 39.6 hours, the student was able to complete the first eight chapters, working between two and three hours daily on the program and stopping between sections and at the end of the chapters to take tests. Since this represents about half of the course, it was estimated that, if students worked on a self-paced basis, they would complete the 17 chapters in approximately 90-100 working hours.

At the end of eight weeks of intensive writing, SMSG Programed First Course in Algebra (Form H) emerged in preliminary form with 1,036 pages, plus key, index, response sheets, and a total of 8,782 items to which the student should respond.

In addition to the preparation of Form H, a form was developed for completion by the teachers for their chapter by chapter evaluation, their evaluation of student response to the program with reference to their ability levels, the estimated time for completion of each section, areas of difficulty encountered, and recommended changes. The 1963 writers also attacked the problem of testing with new vigor. The 1962-63 tests had been criticized by teachers as being too hard, unfair samples of content, and poorly worded. An experienced writer-teacher, Florence Elder, was selected to rewrite the tests for 1963-64. Items were submitted by each of the programers, worked into a 35 minute test, checked by the writers again, and administered to a student subject. The feeling among the writers was that these new tests represent a more characteristic sample of the skills, concepts, and vocabulary on a chapter by chapter basis than the earlier tests.

PLP Centers (1963-64)

In 1963-64, an attempt was made to evaluate Form H in a large scale experiment with Forms CR, MC, 9F, and the SMSG *Introduction to Algebra* using common testing on all groups. The *Introduction to Algebra* text (IA) covers essentially the same material as the *First Course in Algebra* (9F), with simplified language, slower pacing, and more extensive illustrations.

Schools to participate in 1963-64 were selected from those which had participated in 1962-63 and indicated they were interested in continuing, and other schools which had expressed a desire to take part in this SMSG Project. The teachers and students were assigned by the schools to the texts provided by SMSG. The conditions imposed by SMSG were that:

1. all teachers should have had at least one year experience with SMSG *Algebra*,
2. all students were to be in the upper three quartiles on SCAT 2A (School and College Ability Test, Form 2A) to be administered in September 1963, and
3. teachers who were to teach Form CR or Form MC in 1963-64 must have taught these forms in 1962-63.

The first condition was to assure trained teachers, familiar with the content of SMSG *Algebra*, to preclude philosophical disagreement with the subject. The second condition was to assure SMSG that the students were selected from those who would normally take algebra in a precollege course. The third condition was imposed to compare teachers using these materials for a second year with teachers using the conventional text.

In September 1963, all students were given SCAT 2A and the pre-tests PLP-1 and PLP-2. Inadvertently, preliminary forms of the pre-tests were given to some schools and final forms were given to others. Consequently only items common to the two forms were used. Items from the PLP-1 and PLP-2 were lumped together to form a single pre-test score. In TABLE NUMBER 3 the number of students in each mode, as well as the corresponding mean scores on SCAT - verbal, SCAT - quantitative, and the pre-test, are presented.

TABLE NUMBER 3

	N	Mean of SCAT V	Mean of SCAT Q	Mean of Pre-test
MC	266	284.8	301.1	12.5
CR	223	282.7	298.8	10.6
H	642	284.7	298.8	11.2
9F	368	284.2	300.8	11.4
IA	102	275.1	291.8	8.7

For purposes of post-testing, two new tests were constructed: 1) PLP-4 which contained selected items from PLP-2 and PLP-3; and 2) PLP-5 which contained additional selected items from PLP-2 and PLP-3 plus analysis items. Since there has been some conjecture as to whether or not programed instruction is as applicable for teaching higher level skills as it is for teaching computational skills, all items from PLP-4 and PLP-5 were coded as to the level of intellectual activity which they measured. Two scores were generated for each student: 1) Lo-cognitive, measuring intellectual activity equal to or less than manipulating (computation); and 2) Hi-cognitive, measuring intellectual activity greater than manipulating. For a more detailed description of the levels of intellectual activity, see APPENDIX D.

In comparing performance of students using the several modes of presentation, it would be fruitful to examine the interaction of mode and student ability, both verbal and quantitative. That is to say, are there differences in the relative effectiveness of the several modes at different levels of verbal and quantitative ability?

As was stated earlier, the schools were requested to provide students in the upper three quartiles on SCAT, 2A. In fact, as can be seen from the frequencies in TABLE NUMBER 4 through TABLE NUMBER 7, almost all students lie in the upper two quartiles, with the majority of them lying in the top quartile. Unfortunately, this does not provide for a sufficient spread in ability to examine the interaction

TABLE NUMBER 4

QUARTILES ON SCAT VERBAL

		T	Q1	Q2	Q3	Q4
	MC	266 28.8	169 30.5	72 26.1	18 25.4	7 22.3
	CR	223 24.6	134 25.9	63 24.2	24 19.2	2 17.5
MODE	H	642 28.0	407 31.8	147 23.4	66 19.2	22 16.4
	9F	368 28.4	241 30.2	97 26.4	25 21.6	5 19.0
	IA	102 21.3	33 24.1	36 21.4	23 17.0	10 21.5

Cells contain:

1) Frequency
2) Mean (Lo)

TABLE NUMBER 5

QUARTILES ON SCAT VERBAL

		T	Q1	Q2	Q3	Q4
	MC	266 6.1	169 6.5	72 5.3	18 5.4	7 4.7
	CR	223 5.2	134 5.5	63 5.0	24 3.8	2 4.0
MODE	H	642 6.0	407 7.0	147 4.7	66 3.8	22 3.0
	9F	368 5.9	241 6.4	97 5.1	25 3.7	5 2.6
	IA	102 4.2	33 4.8	36 3.9	23 3.6	10 4.5

Cells contain:

1) Frequency
2) Mean (Hi)

TABLE NUMBER 6

QUARTILES ON SCAT QUANTITATIVE

		T	Q1	Q2	Q3	Q4
	MC	266 28.8	145 32.0	91 26.5	23 19.7	7 19.6
	CR	223 24.6	110 28.0	79 22.9	27 17.8	7 16.4
MODE	H	642 28.0	325 32.7	199 25.6	91 20.4	27 15.3
	9F	368 28.4	200 31.4	135 25.7	29 21.2	4 21.5
	IA	102 21.3	31 26.1	32 21.5	30 18.2	9 14.1

Cells contain:

1) Frequency
2) Mean (Lo)

TABLE NUMBER 7

QUARTILES ON SCAT QUANTITATIVE

		T	Q1	Q2	Q3	Q4
	MC	266 6.1	145 7.0	91 5.4	23 4.0	7 3.7
	CR	223 5.2	110 5.8	79 4.9	27 3.9	7 4.0
MODE	H	642 6.0	325 7.3	199 5.2	91 4.2	27 2.8
	9F	368 5.9	200 6.9	135 5.0	29 3.2	4 3.2
	IA	102 4.2	31 5.3	32 4.2	30 3.3	9 3.3

Cells contain:

1) Frequency
2) Mean (Hi)

between mode and ability. Looking at the mean Lo-cognitive and Hi-cognitive scores by mode and ability level, in TABLE NUMBER 4 through TABLE NUMBER 7, indicates that there may in fact be such an interaction. Mode H, for example, seems to become relatively less effective going from higher ability to lower ability. This seems to hold for both verbal and quantitative. The question remains to be answered by research other than this.

Since it was not feasible to compare modes by ability level and since the five groups differed somewhat on ability and even more so on initial performance, a covariance technique was used to give estimates of what the scores on the Lo-cognitive and Hi-cognitive post-test measures of performance would have been had the five treatment groups been comparable on SCAT-verbal, SCAT-quantitative, and initial level of performance. These estimates, or adjusted means, were tested for significance of difference among treatment groups. (See APPENDIX B.)

With the Lo-cognitive post-test measure of performance as the criterion variable, the analysis indicated that the following differences were significant at least at the .05 level:

MC > CR
H > CR
H > IA
9F > CR
9F > IA

While the difference between MC and IA was not significant, it approached significance. The differences between the various combinations of MC, H, and 9F were not significant and neither was the difference between CR and IA significant. Thus MC, H, and 9F tend to cluster together in their effectiveness to teach Lo-cognitive skills. Likewise, CR and IA tend to be equally effective, though significantly less effective than the other three modes.

With the Hi-cognitive post-test measure of performance as the criterion variable, only one difference was found to be significantly different, H > CR. These results tend to indicate that all of the modes are equally effective for teaching Hi-cognitive skills.

In TABLE NUMBER 8, the adjusted means for the Lo-cognitive and the Hi-cognitive measures are presented. It is evident that even for those differences which were significantly different, the differences are small in amount.

TABLE NUMBER 8

MODE	ADJUSTED MEANS	
	Lo-cognitive	Hi-cognitive
MC	27.31	5.68
CR	25.26	5.35
H	28.00	6.01
9F	27.90	5.72
IA	25.70	5.38

While the results involving modes MC, CR, and 9F do not duplicate the results obtained with the 1962-63 data, it must be pointed out that the two years of experimentation were not the same. In the 1962-63 experimentation, teachers used the MC and CR texts for the first time. In the 1963-64 experimentation, only those teachers who had taught the MC and CR texts the previous year were allowed to use them for the following year. The failure of the 1963-64 results to duplicate those of the preceding year might possibly be due to this difference. On the other hand, it might reflect the more sensitive criterion measures and the inclusion of verbal and quantitative ability as covariates.

In February of 1964, and again in May of 1964, all students were administered an opinion questionnaire with respect to the text which they were using. One of the questions in each of the questionnaires dealt with the frequency with which students were required to get help from their teacher. The results of this questionare presented in TABLE NUMBER 9. It can be seen that there is little difference by mode, though the need for help seems to increase later in the year for students in all modes.

TABLE NUMBER 9

Need Help From Teacher	FEBRUARY					MAY				
	MC	CR	H	9F	IA	MC	CR	H	9F	IA
Very often	25	27	24	34	30	38	48	41	47	44
Seldom	61	60	64	56	66	52	43	49	45	47
Not At All	11	11	05	06	02	06	06	03	03	04
No Response	03	02	06	04	02	04	03	07	05	05

FIGURES ARE IN PERCENT

Another three questions in each of the questionnaires dealt with confidence in using "mathematical vocabulary", "mathematical skills", and "mathematical concepts". The results of these three questions are presented in TABLE NUMBER 10 through TABLE NUMBER 12.

TABLE NUMBER 10

Confidence in Using "Mathematical Vocabulary"	FEBRUARY					MAY				
	MC	CR	H	9F	IA	MC	CR	H	9F	IA
Not Confident	16	24	15	16	13	32	34	20	26	37
Fairly Confident	68	65	71	70	81	58	58	69	66	56
Very Confident	15	10	13	13	06	09	07	10	07	06
No Response	01	01	01	01	00	01	01	01	01	01

FIGURES ARE IN PERCENT

Both in February and May, the majority of the students were fairly confident in using "mathematical vocabulary". The differences by mode were greatest at the end of the first semester, ranging from 65% of the students in CR indicating "fairly confident" to 81% in the IA group. The percent in the "fairly confident" category diminished by the end of the year in May, with those in the H group maintaining its percent most nearly the same (from 71% to 69%). This group also lost the least grounds due to "spilling" over into the "not confident" category.

TABLE NUMBER 11

Confidence in Using "Mathematical Skills"	FEBRUARY					MAY				
	MC	CR	H	9F	IA	MC	CR	H	9F	IA
Not Confident	20	30	18	18	22	33	44	27	25	41
Fairly Confident	58	59	60	51	58	51	47	59	61	46
Very Confident	21	10	22	29	19	15	08	13	13	12
No Response	01	01	00	01	01	01	01	01	01	01

FIGURES ARE IN PERCENT

Again, those in the H group seemed to be the most stable in the "fairly confident" category. The two groups which seemed to have suffered most with respect to confidence in using "mathematical skills" were those for constructed response (CR) and introduction to algebra (IA). By the end of the year, the population in the "not confident" and "fairly confident" categories were about the same for these groups--approximately 45% in each category.

TABLE NUMBER 12

Confidence in Using "Mathematical Concepts"	FEBRUARY					MAY				
	MC	CR	H	9F	IA	MC	CR	H	9F	IA
Not Confident	17	28	16	18	22	25	39	25	28	35
Fairly Confident	59	60	65	57	65	60	50	63	60	56
Very Confident	22	11	18	23	13	13	08	11	10	07
No Response	02	01	01	01	00	02	03	01	02	01

FIGURES ARE IN PERCENT

Again it can be seen that there is little difference by mode, though confidence seems to decrease a small amount later in the year.

In interpreting the preceding four tables it is important to recall that there are differences among the modes on ability and pre-test performances. The differences are slight for modes MC, CR, H, and 9F and somewhat larger for mode IA. See TABLE NUMBER 3.

The May administration of the questionnaire asked for agreement or disagreement with the statement, "I would like to have more textbooks like the one we used this year". The results are presented in TABLE NUMBER 13. It is quite interesting to note that mode H is the only mode for which more students agreed than disagreed to the statement. This suggests the possibility that, while several of the modes may be equally effective in presenting the material, mode H is the better liked, possibly less boring.

TABLE NUMBER 13

	"I would like to have more math textbooks like the one we used this year".				
	MC	CR	H	9F	IA
Agree	33	25	51	35	30
Disagree	61	70	43	59	63
No Response	05	04	05	05	06

FIGURES ARE IN PERCENT

In TABLE NUMBER 14, the mean scores on SCAT-verbal, SCAT-quantitative, Lo-cognitive post-test, and Hi-cognitive post-test, by mode and by response to the preceding question, are presented. While there was little difference in verbal or quantitative ability between those students who agreed and those who disagreed with the statement, there was a larger difference in post-test performance.

TABLE NUMBER 14

		"I would like to have more math textbooks, like the one we used this year.				
		MC	CR	H	9F	IA
Agree	SCAT V	284.75	283.55	286.13	284.80	277.35
	SCAT Q	301.33	300.61	301.03	302.61	294.90
	Lo-cog.	31.49	27.00	30.24	30.61	23.42
	Hi-cog.	6.80	5.77	6.56	6.45	4.74
Disagree	SCAT V	285.17	282.39	282.53	283.38	273.08
	SCAT Q	301.29	297.97	295.95	299.83	290.15
	Lo-cog.	27.64	23.66	25.10	27.00	20.17
	Hi-cog.	5.78	4.95	5.35	5.48	3.91

One of the questions in the February questionnaire dealt with the general impression of the course, i.e., favorable - unfavorable. The results of this question are presented in TABLE NUMBER 15. Another measure of attitude toward the course, as taken from unstructured COMMENTS, is presented in TABLE NUMBER 16.

TABLE NUMBER 15

Impression of the Course	MC	CR	H	9F	IA
Very Unfavorable	05	06	02	02	00
Not Favorable	14	22	08	10	11
Neither Favorable Nor Unfavorable	24	23	26	30	35
Favorable	44	39	49	47	47
Very Favorable	13	08	13	10	07
No Response	00	01	01	01	00

FIGURES ARE IN PERCENT

In each mode except in CR, those whose impressions were either "favorable" or "very favorable" were in the majority. Form H accounts for the highest total in these two categories, but it differs only by 5% from MC or 9F.

TABLE NUMBER 16

Attitude Toward the Course	FEBRUARY					MAY				
	MC	CR	H	9F	IA	MC	CR	H	9F	IA
Negative	13	19	11	08	14	27	35	17	17	32
Neutral	21	17	23	22	10	35	28	45	42	15
Positive	24	19	24	24	18	14	09	13	10	07
No Response	42	45	42	46	58	23	28	25	30	46

FIGURES ARE IN PERCENT

Attitudes toward the material appear to be generally favorable. There is little difference by mode except that, throughout the year, mode CR is looked upon somewhat less favorably and, toward the end of the year, mode IA is also looked upon somewhat less favorably. This is in agreement with the results of the main analysis.

In the May questionnaire, two questions dealt with the general impression of the course as regards to interest and difficulty. The results of these questions are presented in TABLE NUMBER 17 and TABLE NUMBER 18.

TABLE NUMBER 17

Impression of the Course	MC	CR	H	9F	IA
Very Easy Most of the Time	01	01	01	00	04
Easy Most of the Time	09	09	13	11	10
Neither Easy nor Difficult	38	31	32	32	29
Difficult Most of the Time	37	38	32	38	33
Very Difficult Most of the Time	06	09	06	07	08
No Response	09	11	15	11	16

FIGURES ARE IN PERCENT

TABLE NUMBER 18

Impression of the Course	MC	CR	H	9F	IA
Very Interesting	07	07	09	08	09
Interesting	29	18	38	40	27
Neither Interesting nor Uninteresting	24	29	24	24	24
Uninteresting	16	17	07	07	12
Very Uninteresting	10	06	05	06	07
No Response	14	23	16	13	20

FIGURES ARE IN PERCENT

There appears to be little difference among the modes on these two variables. In general, the materials tend to be considered somewhat difficult and interesting.

In TABLE NUMBER 19, SCAT-verbal, SCAT quantitative, Lo-cognitive, Hi-cognitive, and perceived difficulty are correlated with interest. On the supposition that the bright students, the high achievers, and those who perceive the course as easy would find the programed materials boring, one would expect to find negative correlations. The results, however, tend not to support this supposition.

TABLE NUMBER 19

	Uninteresting - Interesting				
	MC	CR	H	9F	IA
SCAT-verbal	.02	-.13	.07	.13	.12
SCAT-quantitative	.15	.15	.19	.10	.24
Lo-cognitive	.25	.40	.28	.34	.27
Hi-cognitive	.26	.37	.22	.29	.12
Difficult - easy	.43	.21	.27	.37	.39

FIGURES ARE PRODUCT MOMENT CORRELATIONS

Similarity, ability, performance and perceived difficulty are correlated with the general impression of the course (unfavorable - favorable), as measured in the May questionnaire, in TABLE NUMBER 20. They are correlated with the attitude toward the course (negative - positive), as gotten from unstructured COMMENTS, in TABLE NUMBER 21 .

TABLE NUMBER 20

	Unfavorable - Favorable				
	MC	CR	H	9F	IA
SCAT-verbal	.10	.07	.13	-.05	.08
SCAT-quantitative	.07	.09	.22	.09	.29
Lo-cognitive	.21	.17	.27	.24	.37
Hi-cognitive	.23	.25	.22	.18	.24
Difficult - easy	.35	.18	.29	.34	.31

FIGURES ARE PRODUCT MOMENT CORRELATIONS

TABLE NUMBER 21

	ATTITUDE TOWARD COURSE: negative - positive									
	FEBRUARY					MAY				
	MC	CR	H	9F	IA	MC	CR	H	9F	IA
SCAT-verbal	.01	.00	-.02	.01	.00	-.07	.18	.08	.07	.28
SCAT-quantitative	-.01	.04	.13	.07	.23	-.03	.10	.25	.07	.04
Lo-cognitive	.12	.04	.08	.13	.22	.10	.17	.23	.09	.08
Hi-cognitive	.04	.05	.06	.10	.20	.05	.20	.18	.09	.27
Difficult - easy	.16	.19	.14	.15	.40	.30	.22	.25	.23	.33

FIGURES ARE PRODUCT MOMENT CORRELATIONS

The correlations, while generally quite low, tend to be positive. That is, students who are high in ability, who are high achievers, and who perceive the course as easy have a favorable attitude toward the course.

In order to examine the degree to which performance was dependent on ability, the Lo-cognitive and Hi-cognitive measures were correlated with SCAT-verbal and SCAT-quantitative. These results are presented in TABLE NUMBER 22. In addition, the unstructured COMMENTS were coded for spelling errors and grammar errors. These were correlated with the Lo-cognitive and Hi-cognitive measures. The results are presented in TABLE NUMBER 23 and TABLE NUMBER 24. Mode H seems to be somewhat more highly correlated with SCAT-verbal than are the other modes. All modes are rather highly correlated with SCAT-quantitative. The correlations of the Lo-cognitive and Hi-cognitive measures with spelling and grammar errors tend to be quite low.

TABLE NUMBER 22

	SCAT-verbal					SCAT-quantitative				
	MC	CR	H	9F	IA	MC	CR	H	9F	IA
Lo-cognitive	.34	.29	.59	.41	.32	.57	.54	.60	.53	.62
Hi-cognitive	.30	.23	.52	.42	.19	.49	.40	.54	.49	.44

FIGURES ARE PRODUCT MOMENT CORRELATIONS

TABLE NUMBER 23

	SPELLING ERRORS IN COMMENTS									
	FEBRUARY					MAY				
	MC	CR	H	9F	IA	MC	CR	H	9F	IA
Lo-cognitive	.15	.08	.14	.18	.01	.09	.13	.13	.28	.19
Hi-cognitive	.03	.05	.14	.10	.03	.07	.17	.07	.22	-.05

FIGURES ARE PRODUCT MOMENT CORRELATIONS

TABLE NUMBER 24

	GRAMMAR ERRORS IN COMMENTS									
	FEBRUARY					MAY				
	MC	CR	H	9F	IA	MC	CR	H	9F	IA
Lo-cognitive	.05	.07	.07	.02	.30	.10	-.02	.19	.20	.10
Hi-cognitive	.01	.13	.07	-.02	-.14	.09	-.06	.10	.15	-.05

FIGURES ARE PRODUCT MOMENT CORRELATIONS

In addition to performance, it is of interest to consider the relative coverage of the five modes. In TABLE NUMBER 25 the percent of classes having finished the material through Chapters 14, 15, and 16-17 as of June 15th is presented. While no mode accomplished anything near complete coverage of the text, mode H appears to offer the best coverage, with mode MC second best. These are results similar to those found with regard to performance.

TABLE NUMBER 25

	THRU 14	THRU 15	THRU 16-17
MC	47	26	21
CR	62	15	8
H	55	42	30
9F	23	9	5
IA	29	0	0

FIGURES ARE IN PERCENT

Follow-up Study of Retention (May 1964)

In May of 1964, about 450 students who had been in the 1962-63 classes were administered a retention test (PLP-4) to determine whether the algebra they had studied was related to the mode they had used. These students were regular 9th graders studying on a time basis with a teacher. The students selected were randomly chosen from classes which had used modes MC, CR, or 9F in 1962-63.

Each student was asked to indicate the type of mathematics he was studying the following year. In TABLE NUMBER 26 the number of students, as well as their mean score on PLP-4, is presented for each category of mathematics being studied within each mode.

TABLE NUMBER 26

	MODE		
Type of Math Taken the Following Year	MC	CR	9F
Geometry	147 15.5	106 17.1	39 15.3
Algebra	27 12.9	13 16.7	--
Other	6 7.3	16 7.6	--
No Math	15 8.6	38 9.7	12 9.6
No Response	33 21.7	5 13.3	1 4.0
Total	228 15.4	176 14.6	52 13.8

Cells Contain:

1) Frequency

2) Mean (PLP-4)

For testing the significance of difference among the three groups, a covariance technique was again used. PLP-4 was the criterion variable and PLP-1 and PLP-2, given in September of 1962, were the covariates. The null hypothesis of no difference among treatments after adjusting with covariates was not disproved. (See APPENDIX C.) That is to say, that one year after completing the course, performance on PLP-4 was not significantly different for students who had used different modes, i.e., MC, CR or 9F. In TABLE NUMBER 27 the means of PLP-4, adjusted for PLP-1 and PLP-2, are presented.

TABLE NUMBER 27

MODE	ADJ. MEAN PLP-4
MC	14.90
CR	15.06
9F	14.41

While it might have proved interesting to look at retention in terms of Lo-cognitive and Hi-cognitive performance, PLP-4 contained an insufficiently large enough number of Hi-cognitive items to generate such a score.

Final Revision of the Hybrid (Summer 1964)

During the 1964 SMSG Summer Writing Session, a writing team chaired by D. Blakeslee and consisting of W. G. Chinn, F. Jacobson, M. Matchett, W. Matson, and P. Redgrave comprehensively revised the hybrid text. This revision included significant changes in the mathematical content of 9F based on critical reviews by the PLP writers and analytical comments from members of the SMSG Advisory Board, notably those of P. D. Lax, New York University, and H. O. Pollak, Bell Telephone Laboratories. In addition, a Teacher Commentary was prepared.

During the 1963-64 testing, as each chapter was completed, teachers administered the SMSG-PLP chapter tests and returned the answer sheets for analysis. In support of the revision, a separate random sample of 100 of these answer sheets were pulled for each chapter and mode, and a comprehensive item analysis was prepared. Furthermore, for about 100 students using mode H and about 100 students using mode 9F, a record was kept of the number of errors per item and of the total scores on PLP-1, 2, 4, and 5 and on each of the chapter tests. Finally, teacher reaction questionnaires for each chapter and mode were utilized.

The final revision of the hybrid text concluded SMSG's examination of programed instruction. The original literature on programing does not provide for a program "mix". However, the mixed form tried in the 1964 writing has more clearly met the criteria of improved student performance and greater philosophical satisfaction to the writers. In view of this, an examination of programed materials that are feasible and promising should include such hybridization for consideration.

APPENDIX A

Test Score Estimates

Since the treatment groups are not comparable on the pre-tests, analysis of covariance is appropriate for adjusting the post-tests for the effect of the differences on the pre-tests and for testing the significance of over-all differences among the treatment groups on the post-tests.

Case I PLP-2 (post) is the dependent variable and PLP-1 (pre) and PLP-2 (pre) are the covariates.

The adjusted means are:

CR	16.6580
MC	16.2195
MCR	13.8592
9F	14.8990

The variance ratio is:

F(3, 875) = 279.4771 / 18.9009 = 14.79
CRITICAL VALUE (.001 level) = 5.42

CASE II PLP-3 (post) is the dependent variable and PLP-1 (pre) and PLP-2 (pre) are the covariates.

The adjusted means are:

CR	10.7214
MC	10.9281
MCR	9.2799
9F	9.8927

The variance ratio is:

F(3, 875) = 102.1206 / 12.2316 = 8.35
CRITICAL VALUE (.001 level) = 5.42

The analysis of covariance model assumes homogeneity of variance on the dependent variable. Applying Bartlett's test, we find:

Case I For PLP-2 (post)

$$V = 5.50$$

$$\text{CRITICAL VALUE (.01 level)} = 11.34$$

Case II For PLP-3 (post):

$$V = 52.83$$

$$\text{CRITICAL VALUE (.01 level)} = 11.34$$

Since the assumption of homogeneity of variance does not hold under Case II, caution should be observed in interpreting the analysis, particularly since classes were formed without random assignment. The standard deviations of PLP-3 (post) for the four treatment groups are as follows:

CR	3.82
MC	5.06
MCR	3.50
9F	3.30

Since F, as an over-all test, indicates significant differences among the groups, we may safely make further tests to see whether selected pairs of means differ significantly. This involves the computation of D / S_D for each pair, where D is the difference between the adjusted means and S_D is the standard error of the difference between the adjusted means.

Since we have no a priori hypotheses for making comparisons between selected pairs of means, the t test is inappropriate for testing D / S_D in that we would tend to capitalize on change-large differences and consequently lower our chosen level of significance. Instead, we will use Scheffe's S-method which requires that D / S_D be equal to or greater than K if we are to consider the difference significant, where K is defined as:

$$K = \sqrt{(G-1)\, F_\alpha}$$

with α being the chosen level of significance and df: $n_1 = G-1$ and $n_2 = N-G-2$.

For α = .001, K = $\sqrt{(3)(5.42)}$ = 4.03
= .01, = $\sqrt{(3)(3.78)}$ = 3.37
= .05, = $\sqrt{(3)(2.60)}$ = 2.79

df: $n_1 = 3$ and $n_2 = 875$.

Case I PLP-2 (post) is the dependent variable and PLP-1 (pre) and PLP-2 (pre) are the covariates.

1) CR - 9F: D / S_D = 1.7590 / 0.4628 = 3.80
2) MC - 9F: 1.3205 / 0.4511 = 2.93
3) 9F - MCR: 1.0398 / 0.5199 = 2.00
4) CR - MC: 0.4385 / 0.3602 = 1.22
5) CR - MCR: 2.7988 / 0.4605 = 6.08
6) MC - MCR: 2.3603 / 0.4492 = 5.25

Case II PLP-3 (post) is the dependent variable and PLP-1 (pre) and PLP-2 (pre) are the covariates.

1) CR - 9F: D / S_D = 0.8287 / 0.3723 = 2.23
2) MC - 9F: 1.0354 / 0.3629 = 2.85
3) 9F - MCR: 0.6128 / 0.4183 = 1.46
4) MC - CR: 0.2067 / 0.2897 = 0.71
5) CR - MCR: 1.4415 / 0.3704 = 3.89
6) MC - MCR: 1.6482 / 0.3613 = 4.56

References:

McNemar, Quinn. *Psychological Statistics*. 3rd ed. John Wiley and Sons, New York, 1962.

Winer, B. J. *Statistical Principles in Experimental Design*. McGraw - Hill Book Company, Inc., New York, 1962.

APPENDIX B

Lo-Hi Cognitive Estimates

Since the treatment groups are not comparable on student ability (as measured by SCAT, 2A) or on the pre-test, analysis of covariance is again appropriate for adjusting the post-test measures for the effect of the differences in ability and pre-test performance and for testing the significance of over-all differences among the treatment groups on the post-test measures.

Case I The Lo-cognitive post-test measure is the dependent variable and SCAT-verbal, SCAT-quantitative, and pre-test performance are the covariates.

The adjusted means are:

MC	27.3148
CR	25.2633
H	28.0039
9F	27.9023
IA	25.6955

The variance ratio is:

F(4, 1593) = 402.2607 / 38.2326 = 10.52

CRITICAL VALUE (.001 level) = 4.62

Case II The Hi-cognitive post-test measure is the dependent variable and SCAT-verbal, SCAT-quantitative, and pre-test performance are the covariates.

The adjusted means are:

MC	5.6823
CR	5.3464
H	6.0121
9F	5.7188
IA	5.3821

The variance ratio is:

F(4, 1593) = 23.6080 / 4.264 = 5.50

CRITICAL VALUE (.001 level) = 4.62

The analysis of covariance model assumes homogeneity of variance on the dependent variable. Applying Bartlett's test, we find:

Case I   For Lo-cognitive post-test measure:

$$V = 5.69$$

$$\text{CRITICAL VALUE } (.01 \text{ level}) = 13.28$$

Case II   For Hi-cognitive post-test measure:

$$V = 4.21$$

$$\text{CRITICAL VALUE } (.01 \text{ level}) = 13.28$$

Thus we find the assumption of homogeneity of variance satisfied for both cases.

Since F, as an over-all test, indicates significant differences among the groups, we may safely make further tests to see whether selected pairs of means differ significantly. We could, on the basis of the previous year's results and on theoretical consideration, make specific hypotheses regarding selected pairs of means and test these hypotheses via t tests. The 1962-63 experimentation, however, is sufficiently different from the 1963-64 experimentation that it seems more desirable to make all possible comparison and to treat those parts of the two years' experimentation that are alike as a replication. Testing all possible comparisons makes the t test inappropriate in that we would tend to capitalize on chance-large differences and consequently lower our chosen level of significance. Again we will use Scheffe's S-method which requires the D / S_D (the difference between the adjusted means divided by the standard error of the difference) be equal to or greater than K if we are to consider the difference significant, where K is defined as:

$$K = \sqrt{(G-1)\,F_\alpha}$$

with α being the chosen level of significance and df: $n_1 = G-1$ and $n_2 = N-G-2$.

$$\begin{aligned} \text{For } \alpha &= .001, & K &= \sqrt{(4)(4.62)} = 4.30 \\ &= .01, & &= \sqrt{(4)(3.32)} = 3.65 \\ &= .05, & &= \sqrt{(4)(2.37)} = 3.08 \end{aligned}$$

df: $n_1 = 4$ and $n_2 = 1{,}595$.

Case I The Lo-cognitive post-test measure is the dependent variable and SCAT-verbal, SCAT-quantitative, and pre-test performance are the covariates.

1) H - MC: D / S_D = 0.6891 / 0.4542 = 1.5171
2) H - CR: 2.7406 / 0.4817 = 5.6894
3) H - 9F: 0.1016 / 0.4056 = 0.2504
4) H - IA: 2.3084 / 0.6714 = 3.4381
5) MC - CR: 2.0515 / 0.5659 = 3.6251
6) 9F - MC: 0.5875 / 0.4999 = 1.1752
7) MC - IA: 1.6193 / 0.7373 = 2.1962
8) 9F - CR: 2.6390 / 0.5257 = 5.0199
9) IA - CR: 0.4322 / 0.7468 = 0.5787
10) 9F - IA: 2.2068 / 0.7050 = 3.1302

Case II The Hi-cognitive post-test measure is the dependent variable and SCAT-verbal, SCAT-quantitative, and pre-test performance are the covariates.

1) H - MC: D / S_D = 0.3298 / 0.1522 = 2.1668
2) H - CR: 0.6657 / 0.1615 = 4.1219
3) H - 9F: 0.2933 / 0.1360 = 2.1566
4) H - IA: 0.6300 / 0.2251 = 2.7987
5) MC - CR: 0.3359 / 0.1897 = 1.7706
6) 9F - MC: 0.3650 / 0.1676 = 2.1778
7) MC - IA: 0.3002 / 0.2472 = 1.2144
8) 9F - CR: 0.3724 / 0.1762 = 2.1135
9) IA - CR: 0.0357 / 0.2503 = 0.1426
10) 9F - IA: 0.3367 / 0.2363 = 1.4248

APPENDIX C

Estimates of Retention

Since the treatment groups are not comparable on the pre-tests (given in September of 1962), analysis of covariance is appropriate for adjusting the post-test (given in May of 1964) for the effect of the differences on the pre-tests and for testing the significance of over-all differences among the treatment groups on the post-test.

PLP-4 is the dependent variable and PLP-1 and PLP-2 are the covariates.

The adjusted means are:

MC	14.90
CR	15.06
9F	14.41

The variance ratio is:

F(2, 451) = 8.6077 / 21.2961 = 0.40
CRITICAL VALUE (.001 level) = 6.91
CRITICAL VALUE (.05 level) = 2.99

The analysis of covariance model assumes homogeneity of variance on the dependent variable. Applying Bartlett's test, we find:

V = 8.41
CRITICAL VALUE (.01 level) = 9.21

Thus we find the assumption of homogeneity of variance satisfied.

APPENDIX D

Levels of Intellectual Activity

1) KNOWING - Knowing terminology, facts, and rules

2) TRANSLATING -
    1. Changing from one language to another
    2. Expressing ideas in verbal, symbolic, or geometric form
    3. Codifying patterms

3) MANIPULATING (Computation) -
    1. Carrying out algorithms
    2. Using techniques

4) CHOOSING -
    1. Making comparisons
    2. Selecting appropriate facts and techniques
    3. Guessing
    4. Estimating
    5. Changing one's approach
    6. Selecting new symbolism

5) ANALYZING -
    1. Analyzing data
    2. Finding differences
    3. Recognizing relevant and irrelevant data
    4. Seeing patterns, isomorphisms, and symmetries
    5. Analyzing proofs
    6. Recognizing need for additional information
    7. Recognizing need for proof or counterexample

6) SYNTHESIZING -
    1. Specializing and generalizing
    2. Conjecturing
    3. Formulating problems
    4. Constructing a proof or a problem

7) EVALUATING -
    1. Validating answers and the solution process
    2. Judging reasonableness of answers
    3. Criticizing proofs
    4. Judging the significance of a problem

APPENDIX E

Sample Summary Guide For Writing

Chapter 12 - Polynomial and Rational Expressions

In this chapter, the concept of a polynomial is introduced and it is seen that the problem of factoring polynomials is most interesting when restricted to factors which are themselves polynomials. Most of the work is with polynomials over integers i.e., polynomials whose coefficients are integers. Polynomials over the rational and real numbers are also considered. Each of these sets of polynomials is closed under addition and multiplication.

The problem of factoring is to write a given polynomial, which is considered to be of a certain type, as an indicated product of polynomials of the same type. A polynomial which is not factorable as a polynomial over the integers may or may not be factorable when regarded as a polynomial over the real numbers. Factoring a polynomial over the rational numbers can be reduced to factoring a polynomial over the integers.

It was found that factoring is a useful tool for solving equations. The various methods of factoring are all just applications of the distributive property. Besides immediate change of an indicated sum into an indicated product, we have the difference of squares, perfect squares, completing the square, and quadratic polynomials in one variable. The technique for factoring quadratics is based strongly on the knowledge which has been obtained about factors of numbers, and minimizes guess work.

The concept of a rational expression is then considered and it is observed that rational expressions have the same relationship to polynomials as rational numbers have to integers. Problems of simplifying rational expressions are similar to the problems for rational numbers. It is seen that the rational expressions have the usual properties of fractions and that factoring of polynomials plays the same role in the work with rational expressions as factoring of integers plays in the work with rational numbers.

Every rational expression can be written as an indicated quotient of two polynomials which do not have common factors.

A systematic method for division of polynomials in one variable is developed. This is based on the following important property of polynomials: For any two polynomials N and D with D different from zero, there exist polynomials Q and R, with R of lower degree than D, such that $N = QD + R$. The process of division gives us a way of calculating Q and R when N and D are given.

APPENDIX F

Manual for Programers

TABLE OF CONTENTS

Page

1-0 Introduction 47
1-1 Historical Background 47
2-0 The Job Ahead 50
2-1 Content Analysis 51
2-2 Specification of Goals 52
2-3 Idealization of the Learning Sequence 52
2-4 Writing a First Draft - Frames 54
2-5 Revision: Comments and Criticisms 73
2-6 Revision and the Second Draft 74
2-7 Student Try-out: Small and Large Scale Evaluation 76
3-0 Program Analysis 77
3-1 Criterion Items 77
3-2 Flow Charts and Concept Matrices 78
3-3 Logical Analysis 78
3-4 Data: Collection and Reduction 79
Bibliography 81

Appendices

1 Addenda Based on 1964 Experience 82
2 Sample Chapter Summary 86
3 Sample Summary of Chapters 88
4 Sample Objectives 91

PREFACE

In 1961, the School Mathematics Study Group decided to undertake a project in programed learning with special reference to the kind of mathematics illustrated in the texts prepared by the SMSG. The Project has been under the supervision of a Panel consisting of: R. C. Buck, University of Wisconsin; E. E. Hammond, Jr., Phillips Academy (Andover); L. D. Hawkinson, San Francisco Public Schools; J. G. Holland, Harvard University; W. J. McKeachie, University of Michigan; H. N. McNeille, Case Institute of Technology; H. O. Pollak, Bell Telephone Laboratories; and D. W. Taylor, Yale University. The Project has been coordinated by Leander W. Smith, Assistant to the Director of the SMSG, E. G. Begle.

The Panel decided as a first step to have the mathematics of the SMSG First Course in Algebra translated into programed form. Two program formats were chosen by the Panel in 1961: (1) The first of these consisted of a program based on the constructed response linear programs developed by B. F. Skinner, Harvard psychologist; (2) The second of these consisted of a program based on the scrambled book, multiple-choice (intrinsic) programs developed by N. Crowder of U. S. Industries. In the spring of 1963 a hybrid format was proposed by the coordinator which attempted to combine the features of both of the earlier programs and retain the content of the First Course in Algebra. The latter program was revised in the summer of 1964 for release to schools in the fall of 1964.

In preparing the programs, the usual SMSG procedures were followed. The programing was done by mathematicians, teachers of mathematics, and psychologists working together on writing teams. The materials were tested in a variety of classroom situations and revised on the basis of the experience gained. Such materials as prove effective will be made generally available; however, perhaps more important will be any explicit and detailed information on programing procedures, classroom experiences, techniques for using programs which will be made available from time to time as the project progresses.

During the summer of 1961, a Manual was prepared by the project coordinator to be used as a basic document in this effort. It was designed to supply the individuals who were already in command of the mathematics with the information that they would need for the actual construction of programed texts. The present Manual contains much that was in the earlier version and, in addition, draws heavily on examples of programing done in the 1961-64 years of the Project. As programed learning evolves from its earlier stages, so too this Manual will need to reflect and anticipate further changes. Suggestions for the improvement of this Manual should be sent to: SMSG Programed Learning Project, SMSG - Stanford University, Stanford, California.

1-0 Introduction

This Manual is written for programers (and potential programers) whose specific assignment is the preparation of programed texts in mathematics based on the work of the SMSG. Many problems are presented and topics discussed which have failed to appear in other sources; their appearances here will be justified if they shed light on the psychology behind and the methodology of programed instruction.

1-1 Historical Background

The storm and wild enthusiasm experienced in the past few years over "automated teaching" has some of its roots in the classical tradition. From the Platonic dialogue came the Socratic approach of directing students to desired goals by asking questions; from the colonial hornbook came the incremental step approach; and from many sources came the practice of rewarding students who mastered their lessons. But programed instruction transcends simple reward techniques and increments of rote learning; programed instruction is proposed as an application in the classroom of a theory developed in psychological laboratories (see J. G. Holland: "Teaching Machines: An application of Principles from the Laboratory" in Lumsdaine and Glaser: Teaching Machines and Programmed Learning: A Source Book, pp. 215-29) and self-instruction and trouble-shooting techniques in military training bases.

In 1926, Sidney Pressey (see Lumsdaine and Glaser, op. cit., pp. 32-51 and pp. 69-88) developed a self-scoring device to test students without consuming the time and energy of an instructor. He anticipated the device would be useful as a teaching instrument; but in 1926 the idea did not take hold and little was done to develop the self-instructional devices for almost thirty years.

With the publication of B. Frederick Skinner's articles on "The Science of Learning and the Art of Teaching" (see Lumsdaine and Glaser, op. cit., pp. 99-113) in 1954 and "Teaching Machines" (see Lumsdaine and Glaser, op. cit., pp. 137-72) in 1958, there followed what many have called an "industrial revolution in education". Skinner, a Harvard psychologist, presented the notion of operant conditioning as the psychological basis for a type of programed learning. Although many efforts reproduce the Skinnerian programed format, they tend to neglect the essence of his psychology--the shaping of behavior operant (rather than reflex) conditioning. To ensure activity and participation of the learner, the Skinner-type program requires the student to construct his answers. The Analysis of Behavior, a text in psychology by B. F. Skinner and J. G. Holland, provides an illustration of the Skinner-type format which has been widely used.

While Skinner and his followers continued to investigate constructed responses, frequency of response, confirmation mode, overt vs covert responses, error rates, and time variables, the multiple-choice mode evolved. Norman Crowder (see Lumsdaine and Glaser, op. cit., pp. 286-98) added the corrective element and developed "intrinsic programing" which presents a topic to a student with a multiple-choice question for which he selects one alternative. If he has chosen a correct response, he continues along a "main line" program; if he has chosen an incorrect or less correct response, he may receive further exposition, enrichment, or be returned to the topic before proceeding along the main line. Electro-mechanical machines are capable of handling this type of program, although a scrambled text may be as effective.

Teaching machines, as such, shall not be discussed in detail here since their role has not been clearly defined at this time. It is not the intent nor purpose of this manual to criticize the manufacturers of specific machines or programs, but simply to call attention to their work as it has acted as a stimulant to the SMSG Project. For a partial listing of available machines and programs see Rigney and Fry: Audio-Visual Communications Review Supplement Three (May-June 1961) or Finn and Perrin: Teacning Machines and Programed Learning (1962 TDP of USOE).

Simple machines include: sliding masks, paper-fed machines, roll type machines, punchboards, and variations of these. More complex mechanisms include: electric typewriters, audo-visual units, and computer-based machines. The role of the computer is not yet fully known. Electronics corporations are currently engaged in research to investigate educational programing. What is likely to happen in the next decade is open to speculation. At this state, it is obvious that speed of operation and the variety of display potential are but two of the contributions which a computer-based system can offer.

While machines and hardware occupy the labors of some, many publishers are engaged in the production of texts in "programed form". Many, if not most, appear to be sterile reproductions of early programing efforts. Others are reflecting the widespread programing studies which have sprung up in colleges and universities throughout the country with subject matter ranging from statistics to foreign language training. Extensive programing of mathematics by persons expert in programing but not in mathematics has led SMSG to begin its study with persons thoroughly acquainted with the mathematics in question and to teach these people to program.

The call to research has been voiced from many critics and historians of the programed learning movement. Among them, Wilbur Schramm (in Programed Instruction Today and Tomorrow, pp. 73-4) comments:

> "...When we understand more than we do now about how to combine different kinds of programing, how to vary the schedule of reinforcement, and how to fit a program to different learning objectives and student abilities, above all when we learn more about how to maintain the student's interest and challenge him through what is now very often a very dull exercise in conditioning, then programs are likely to look far different from the way they look today."

2-0 The Job Ahead

Programing is not an easy task! It requires a thorough knowledge of the subject, a sensitivity to the ability of the student for whom the material is being prepared, and an extraordinary patience. The pearly prose written on Monday may be torn apart on Tuesday by a student's "I don't understand." The time spent writing enough to keep a student busy for one class hour may range to five or six hours, not including editorial and revision time spent. Is programing an art, or is there a semi-systematic procedure by which the writing can be completed?

Let us examine briefly the procedures used by SMSG in programing the First Course in Algebra.

(1) Content analysis - chapter summaries were prepared by members of the original writing team to give programers an idea of what is covered, why it occurs at a given time, and what review of material seems required.

(2) Statement of Objectives - general objectives and sample questions to measure the completion of these objectives were prepared (see APPENDIX 4). In addition, each programer was required to specify the objectives of the section being programed and to list these at the beginning of the draft version. These section-by-section objectives could then be checked in the reviewing process.

(3) Preparation of a first draft by a programer - the first draft of a given section was required to preserve the content and sequence of the original text. Often an outline of the section was made prior to the writing of material. This stage may appear on cards although ditto was more effective and satisfactory.

(4) Review and criticism by other programers - all members of the team reviewed all of the first several sections to gain uniformity of writing and style and see variety of techniques of presenting ideas.

(5) The "Hearing" - a session at which individual programers defend their writing to the group and offer suggested revisions - semantic, schematic, content, etc.

(6) A revised draft is prepared in the style desired.

(7) A student is found to go through the section to offer criticism and suggestions.

(8) The team captain reviews the draft, offers minor changes, and marks one copy for final typing.

(9) The final copy is prepared for photolithography and the text is printed.

(10) The text is used in a variety of classroom situations with reports to the SMSG on each chapter.

(11) Revision of the text takes place based, in part, on teacher reactions and student performances on SMSG tests of content.

From the experience gained in the summer of 1961, 1962, and 1963, we can abstract a model of how programing might take place:

(1) content analysis
(2) specification of goals
(3) idealization of the learning sequence
(4) construction of the first draft (usually by a mathematician)
(5) revisions and suggestions (usually by teachers)
(6) incorporation of revisions into second draft
(7) student tryout
(8) editing
(9) preparation of final copy

But these steps do not seem to differ considerably from the way that a conventional text is written. The programing of the first draft, however, must be a point of difference at least. In the sections which follow, let us examine the steps (1-9 above) in greater detail with some questions to guide authors in writing future programs in mathematics.

2-1 Content Analysis

"Clearly the programer knows what he's doing." This may be, but more often than not, he fails to see it in writing until after the task is done. This first step in programing is not particularly easy since it involves stating precisely what the material to be covered really is. What terms, concepts, and skills are to be mastered? What, if any, order is explicit in the development of the concepts and/or skills? What order is implicitly agreed upon by other subject area specialists as necessary to coverage of the topics? Will an outline of the content reveal a subject-oriented order of the content? What topics are assumed as backgrounds to the new topic being developed? What subdivision of the content is necessary?

2-2 Specification of Goals

Once the content is clearly defined there is need for explicit statement of goals. What should the student be able to do when he has studied this unit? What terms should he have mastered? Does mastery mean "ability to recognize," "ability to spell, write, or use," or "ability to reconstruct (as in a proof)"? What is the level of the student at the beginning of the program? What tools, terms, concepts does he have to work with when he starts? Which of these shall he need to have refreshed? What item can we present which will test mastery of that which we want to teach?

Content goals must be explicit. For example, we may wish a student to recognize a proof of a theorem, or to perform certain manipulations, or to reproduce from memory a statement generalization, or to factor a given expression over the real numbers. We may want him to generalize from cases or find a specific instance of a given generalization. These all involve the student in an activity toward some specific behavioral goal. It would be of little value to provide only general objectives in terms such as "know how to solve equations," or "understand the number system," or "appreciate the elegance of the proof."

The teachers' commentaries for SMSG texts provide sample test questions to illustrate the specific skills, concepts, and vocabulary desired as outcomes of studying SMSG mathematics. Although these are useful as a guide, they represent only a sampling of the myriad of abilities which a student acquires during an academic year. This statement may seem too obvious but programers will do well to bear in mind that a program may serve in lieu of the combination of text and teacher in the conventional classroom. With this in mind let us move to phase three of the programing process.

2-3 Idealization of the Learning Sequence

Given a topic, a well-defined student, and time, we now come to the stage in programing where teaching experience weighs heavily. A graduate student may have a thorough mastery of elementary material and may (and often does) fail to communicate to his students for any number of reasons. Among the experienced teachers there tends to be a general agreement on the manner of presenting new material to students--agreement in the sense that they can spot (without a student present) the areas that will cause trouble and/or prevent learning.

When should a given topic be introduced? What can be done to motivate the learning? Would an illustration help? What prior knowledge can be assumed? What earlier skills need reinstatement before this topic can be developed? What would a master teacher do to insure learning of the topic?

Idealizing the teaching sequence might go through the following steps:

(1) Review prerequisite concepts through discussion and/or warm up exercises. This may involve presenting a flow chart of earlier ideas and definitions leading to the new topic.

(2) Ask a criterion question to determine readiness for the new topic.

(3) Present items which force discrimination between previous learning and the new topic.

(4) Cite examples or instances of the new topic.

(5) Utilize diagrams, visual aids, and any available media to refine the new concept.

(6) Determine the level of generalization which the student can and should reach. Do not attempt to take all students to equal depth on a topic. A branch in the program may be developed to aid the less able or more able student; a reference to material outside the text may serve the same purpose.

(7) Work through some illustrations. Call for student activity from simple to complex cases.

(8) Test against the original criterion.

(9) Reteach or return the student to material he failed to grasp.

Looking at this from a student's viewpoint may take a similar form:

(1) What earlier material must I recall?
(2) How does the new concept fit into my knowledge?
(3) Am I ready to learn the new topic?
(4) Let's try a few examples.
(5) What counterexamples can I find?
(6) Can I find some original applications?
(7) What test can I use to be sure I know?
(8) What should I go back over?

The basic element to be considered in this planning stage is--the student can learn only if there is material to learn and he will learn only as well as the material is presented. The more careful the planning, the better the first draft is likely to be.

On motivation, let a few words be said. The early programing efforts tended to believe that if a student was correct most of the time (95 percent was the golden number), he would continue to want to learn. This may be, but low error rate cannot be taken alone as an indicator of high learning and motivation. Students in the constructed response format text complained of boredom as much as those in the multiple-choice format text. Hence in 1963, a variety of programing was maintained and the complaints diminished.

2-4 Writing a First Draft

Up to this point we have been concerned with the planning stages. In this section we shall find it necessary to refer to "frames" and "panels". These terms are peculiar to programed texts and will be left almost undefined for the moment to see if, in fact, we can build a flexible attitude and an operational definition as an outcome of our work.

Asking a question, we may require a student to construct his response. For example, we may ask

What is the common name for $(-3) + 4$?	1

The question is enclosed by a frame and the confirmation of the student's response is given immediately to the right. Let us examine other frames and notice the manner of posing questions and the location of confirmations:

Look at the following sentences:

$$5(0) = 0$$
$$3(0) = 0$$
$$0(0) = 0$$
$$m(0) = 0$$

Each time that we find the product of a number and zero our result is zero. Consider the sentence, $k(0) = 0$. Is there a number k which will make this sentence false? ______	No
The sentence $k(0) = 0$ is true for every ______ k.	number, or value of
This property of numbers is called the ______________ __________ of 0.	multiplication property
Simply stated: Any number times _____ equals _____ .	0, 0

The frame above follows a parallel treatment of the multiplication property of 1 and precedes the use of the commutative property to show that: For every number a, $a(0) = 0$ or $(0)a = 0$.

The following frames show how a student may be led through a sequence of items to tie together some of the notions he has studied.

Recall that $\frac{4}{6}$ reduced to _________ terms is $\frac{2}{3}$.	lowest
In fact, $\frac{2k}{3k}$ reduced to lowest terms is _______ .	$\frac{2}{3}$
This is an instance of the general statement that $\frac{ak}{bk} = \frac{\square}{\square}$, for numbers $a, b, k,$ with $b \neq 0$ and $k \neq 0$.	$\frac{a}{b}$
In arithmetic, we noted that $\frac{2}{3} \times \frac{5}{7} = \frac{\square \times \square}{3 \times 7}$.	$\frac{2 \times 5}{3 \times 7}$
In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{\square}$	$\frac{ac}{bd}$
Note that $\frac{a}{b} \times \frac{k}{k} = \frac{a\square}{\square}$, and, just as $\frac{3}{3}$ is another name for 1, $\frac{k}{k}$ is another name for _____.	$\frac{a}{b} \times \frac{k}{k} = \frac{ak}{bk}$ 1
We seen then, that $\frac{a}{b} \times \frac{k}{k} = \frac{a}{b} \times 1 =$ ______ .	$\frac{a}{b}$

②

Together, the definition of multiplication of rational numbers, the fact that $\frac{ak}{bk} = \frac{a}{b}$, and the multiplication property of 1, give us an effective procedure for dealing with complex fractions in arithmetic and algebra. For example, we may use these to find a common name for:

$$\frac{\frac{2}{3} + 5}{\frac{4}{7}}$$

Using the multiplication property of 1:	$\frac{\frac{2}{3} + 5}{\frac{4}{7}} = \frac{\frac{2}{3} + 5}{\frac{4}{7}} \; \frac{21}{21}$	
(We chose $\frac{21}{21}$ because 21 is the least common multiple of ______ and ______ .)		3, 7
We can write:	$\frac{\frac{2}{3} + 5}{\frac{4}{7}} = \frac{\left(\frac{2}{3} + 5\right)(21)}{\Box \; (21)}$	$\frac{4}{7}$
Using the distributive property:	$= \frac{\frac{2}{3}(\Box) + 5(\Box)}{\frac{4}{7}(21)}$	$\frac{\frac{2}{3}(21) + 5(21)}{\frac{4}{7}(21)}$
Multiplying:	$= \frac{14 + \Box}{12}$	105
and adding:	$= \Box$	$\frac{119}{12}$

Some frames require the student to select from alternatives which follow a multiple-choice question, as in the following cases:

Example:

Write the phrase $(5 + a)(a + 8)$ as an indicated sum:

[A] $40 + a^2$

[B] $5a + a^2 + 40 + 8a$

[C] $a^2 + 13a + 40$

Your work might look somewhat like this

$$\begin{aligned}(5+a)\ (a+8) &= (5+a)a + (5+a)8\\ &= 5a + a^2 + 40 + 8a\\ &= a^2 + a(5+8) + 40\\ &= a^2 + 13a + 40\end{aligned}$$

Although on the second line above we have shown the same indicated sum as in [B] above, a better choice is [C].

Example:

41	If you know that the product of two numbers is 0, and that one of the numbers is 3, the other number is ______ .	0
42	If the product of two numbers is 0, at least one of the numbers is ______.	0

43 In Items 41 and 42, does the multiplication property of 0 provide complete answers to the questions?

[A] yes [B] no

[A] You have made the wrong choice. Read [B] .

[B] Another property is required here. It is stated:

If $ab = 0$, then a is 0, or $b = 0$, or both a and b are 0. We shall return to this property in Chapter 7.

16 One pair of parentheses may lie within another. For example, what do you think is the correct common name for $(((3 + 4) \times 5) - 1) \times 2$?

[A] 33
[B] 60
[C] 68
[D] None of these

The correct choice is [C]. The steps that you followed should be those in Items 17 - 22 below. If you chose correctly and are certain of your method, you may skip Items 17 - 22.

Example:

66 Solve the equation $6y + 12 = 2y + (-8)$, writing your work carefully. Then choose the response that describes your method and check the completed solution.

[A] I began by adding (-12) to both sides.
[B] I began by adding $(-12) + (-2y)$ to both sides.
[C] I began by adding $(-2y)$ to both sides.

[A] Fine. You obtain

$$6y + 12 + (-12) = 2y + (-8) + (-12)$$

The truth set is $\{-5\}$.

Check: If y is (-5), the left side is $6(-5) + 12$ or -18, and the right side is $2(-5) + (-8)$ or -18.

[B] Fine. You obtain

$$6y + 12 + (-12) + (-2y) = 2y + (-8) + (-12) + (-2y)$$

The truth set is $\{-5\}$.
Check: If y is (-5), the left side is
$6(-5) + 12$ or -18, and the right side is
$2(-5) + (-8)$ or -18.

[C] Fine. You obtain

$$6y + 12 + (-2y) = 2y + (-8) + (-2y)$$

The truth set is $\{-5\}$.
Check: If y is (-5), the left side is
$6(-5) + 12$ or -18, and the right side is
$2(-5) + (-8)$ or -18.

Example

42 The least common denominator we want is the least common multiple of 4, 10, 45, and 6, which is

[A] $4 \times 10 \times 45 \times 6$
[B] $2 \times 3 \times 5$
[C] $2 \times 2 \times 3 \times 3 \times 5$

The correct choice is [C]. If you made another choice or if you were unable to decide at a glance, complete Items 43 to 53; otherwise, go on to Item 54.

Example:

68 If a number is divisible by 6 and 3, can we conclude that it is divisible by 18?

[A] yes [B] no

If a number is divisible by 6, its prime factorization contains 2×3. Knowing that it is divisible by 3 gives no additional information. Hence, [B] is correct. Notice that 24 is divisible by 6 and 3, but not by 18.

Example:

In Chapter 12 we first studied quadratic polynomials, that is, polynomials in one variable which are of degree two. Remember that in order to find the degree of a polynomial in one variable we first reduce it to common polynomial form. The degree is the highest power of the variable that occurs in any term when the polynomial is in common form.

1 Which of the following are not quadratic polynomials over the real numbers?

(I) $4 - 2x - x^2$

(J) $(3^2)x - 4$

(K) $(2 - x)(2x + 1)$

(L) $x(3x + 2)^2$

(M) $(2x + 1)^2 + 3 - 4x^2$

(N) $\frac{1}{4}x^2$

[A] L

[B] J, N

[C] J, L, M

[D] K, M, N

Although it involves the square of 3, (J) does not involve the square of the variable and is therefore not a quadratic polynomial.

If you put (L) in common polynomial form, you will see that it is of the third degree. The monomial involving the highest power of x is $9x^3$.

(M) is a polynomial of degree 1. Verify this by performing the indicated squaring operation and collecting like terms.

[C] is the correct answer.

Example (confirmation of response followed by further development in text proper):

Yes, the set { -7, -2, 0, 1, 4, q, r, s} contains additive inverses of all the elements of set A = { -4, -1, 0, 2, 7}. The set you have chosen also contains the elements q, r, and s. Do you think that q, r, and s might also be inverse of some of the elements in set A?

We know that sometimes there is more than one number which will satisfy a given condition. For example, both 3 and (-3) have an absolute value of 3. We are now interested in finding out whether there is more than one inverse for a given number.

Let us take the number 3 and assume that an additive inverse of 3 is the number z. Then we can write: $3 + z = 0$. One value of which makes this open sentence true is -3.

Based on what we have studied, is there another value of z which makes $3 + z = 0$ a true sentence?

[A] yes [B] no [C] I don't know

[A] If you think that there is a number other than (-3) which, when added to 3 will give 0, perhaps you can suggest what that number is. If you can't, it might throw some doubt on whether such a number exists. Continue with [C] below.

[B] You may believe that there is no number other than (-3) which added to 3 will give 0 because you cannot think of such a number. This is a likely guess, but can you be sure? Continue with [C] below.

[C] It is true that at this point, we have seen no conclusive evidence whether or not there is a number other than (-3) which will satisfy this equation.

> Since it seems doubtful that there is more than one additive inverse of 3, how can we be absolutely sure? We can use the properties of addition and the definition of the additive inverse to prove that there is only one additive inverse for each number. Let us verify it first for a particular number, 3...

With these examples, it should be clear that frames vary in size and purpose. Some contain cues to the response sought while others depend on recall or parallelism to other material. The number of responses within a frame may vary considerably, too. Some frames are used to check understandings, others to anticipate new material.

There was a tendency in earlier programing efforts to fragment material for the sake of breaking it into small steps. The assumption that behavior was shaped in this way led many of us to sterilize our language and avoid coherence lest it make the step too great for a student. Only as we began to see proofs failing for lack of direction, exercises performed mechanically without a pattern forming, and page after page of "dull, boring, trivia", did we realize that mathematics should not be programed restrictively in frames of three lines' length with one response per frame. In the 1962 version of the SMSG Programed First Course in Algebra Form CR, (p. 6-7) we find the following frames:

50	There are four whole numbers between 5 and 10, namely the numbers 6, 7, 8, and 9. There are ________ (how many) whole numbers between 5 and 9?	3
51	There are ________ (how many) whole numbers between 5 and 8?	2
52	There are ________ (how many) whole numbers between 5 and 7?	1
53	There are ________ (no, some) whole numbers between 5 and 6?	no
54	The set of even numbers which are greater than 5 and also less than 9 contains ________ element(s).	2
	Finally...	
59	The set which contains no elements is called "the empty set" (or "the null set"). Since the set of whole numbers between 5 and 6 does not have any elements in it, it is the ________ set.	empty

As experiences in writing was gained, frames grew larger, incorporating examples, proofs, or topics within and often requiring more than one response. Some dependence on earlier material may be noted in the following pair of frames:

Earlier in this course we solved equations such as $x^2 \div 6x + 5 = 0$. It would be interesting to see what relations exist between the ________ set of $x^2 + 6x + 5 = 0$ and the graph of $y = x^2 + 6x + 5$. with reference to the same set of axes, draw the graphs of:	truth
$y = x^2 + 6x + 5$ $y = x^2 + 6x + 9$ $y = x^2 + 6x + 13$ ______ ______ ______	see response key page xx.
Notice the parabolas $y = x^2 + 6x + 5$ $y = x^2 + 6x + 9$ $y = x^2 + 6x + 13$	
(1) All have the same shape and the same axis of ________ .	symmetry
(2) However, their vertices are (-3, -4) , (-3,), and (,) respectively.	(-3,0), (-3,4)
The curve $y = x^2 + 6x + 5$ appears to intersect the horizontal axis in the two points (, 0)	(-1, 0)
and (, 0).	(-5, 0)
The curve $y = x^2 + 6x + 9$ appears to intersect the horizontal axis in the single point (,).	(-3, 0)
The curve $y = x^2 + 6x + 13$ ________________ (does, does not) intersect the horizontal axis.	does not

Just as there was a tendency to fragment the content, so too there was a tendency to force the same response over and over again. The notion of reinforcement was somehow confused with use, repetition, and variety of presentation. But this is a far cry from the tendency to claim "it is obvious that..." or "it follows simply..."

Periodically, programers, teachers, and students expressed a need for items for which the responses were not immediately confirmed. In the summer of 1963, review sections of the course were presented with problem sets having answers in a key at the back of the book. In addition, certain optional frames and other items where a check point or criterion item seems necessary, were programed in this way. Consider the following:

> Solve the following equations by squaring. The answers are on page xx. If when you have finished you find all of your answers are correct, go on to Item 76; if you have difficulty or wish to check your work, go on with Items 57-76.
>
> 52 $\sqrt{2x} = 1 + x$
>
> 53 $\sqrt{2x + 1} = x + 1$
>
> 54 $\sqrt{x+1} - 1 = x$ Hint: write first as $\sqrt{x+1} = x + 1$
>
> 55 $\sqrt{x} - 1 = x - 7$

In addition to giving hints and complete solutions, it is possible for a program to lead a student into depth outside the text he is using. Often a teacher will provide some students in a class with extra work for enrichment; so then the program can tell the student: "If you had no difficulty with this and would be interested in other cases, read the following pages in..." . An inexpensive booklist is available from SMSG in Newsletter 17 from which references can be taken. In the 1964 revision of Form H, a topical reference guide to the New Mathematical Library series has been incorporated within the text.

Check points are often needed to be sure that students are performing on test-like questions as well as they are performing on individual frames. While this is frequently done by short quizzes in class, the programed text can build these in as a series of self-tests. In the summer of 1963, a self-test was written in Chapter 12 (Section 6b) with answers in the back of the text and references to the earlier section on which the question depends. The student can thus return to topics with which he had trouble or which he failed to master the first time through.

There are many ways in which students can be cued or prompted to give the correct response to an item. Although out of context they seem too obvious, they are frequently used to elicit a student response early in the program.

Underscoring - often used the first time a new word is used or to emphasize a word or phrase; e.g.

A startling development occurs if we consider an original set which is an infinite set.	
N = {1, 2, 3, 4,...} is an _________ _________.	infinite set
E = {2, 4, 6, 8,...} is also an ______ _______,	infinite set
and it is also a proper __________ of N.	subset

Response blank cue - used when we provide a student with information on style of response, when only a part of the response has been worked on, or when data is being accumulated, e.g.

Solve the inequality: $x + (-2) > -3$.	
$(x + (-2)) + 2 > \underline{-3 +}$ __________	-3 + 2
and $x + ((-2) + 2) = x +$ __________	0
so $x >$ __________	x > -1

Note that the student is given part of the response desired, often to make mathematical sense, as in the following

$\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{\square}$	ab

Which would be improved by subsequently calling for $\frac{1}{ab}$ since the reciprocal of ab is what is desired, not ab.

Fading cue - gradual reduction of the stimulus to elicit a given response (spelling is a good example but problem solving can also be done in this way); e.g.

13 × 13 is 169. Therefore 13 is a sq---- r--- of 169.	square root
15 × 15 is 225. Therefore 15 is a _____ ____ of 225.	square root

Choice cue - under the blank a choice may be given to a student to limit his choice to one of two or three words or symbols; e.g.

If 2 is not a factor of y, then y is __________. (even,odd)	odd

or

If x is to the left of y on the number line, then x __________ y. (<, = , >)	$x < y$

Parallel cue - in the fading cue above, the items are written in parallel style; that is, the wording is similar from one item to the next. The following examples are common:

Just as $x + 3$ is the same as $3 + x$, so too, $a + 5$ is the same as ______.	$5 + a$

$\emptyset$ is a subset of $\{0\}$ True or False? _____	True
$\emptyset$ is a _____ of $\{0,1\}$	subset

Empirical cues - exercises in which the student counts, reads, or performs some act to enable him to respond correctly; for example,

If A = {0, 1, 2}, then A is not empty; A contains ________ (how many) elements.	3
If B = {0, 1}, then B contains ________ elements.	2
If C = {6}, then C contains ________ elements.	1
If D = {0}, then D contains ________ elements.	1
Since D = {0}, D is not the empty set.	

or

See if you can complete the following table and discover a rule for finding the number of subsets in any finite set:

number of elements	1	2	3	4	5	6	...	n
number of subsets	2	4	8				...	
number of subsets in exponent form*	2^1	2^2	2^3				...	

* also may be written in expanded form $2^1 = 2$, $2^2 = 2 \cdot 2$, etc. When you finish, compare your table with that shown on page i.

or

Take a penny, a nickel, a dime, a quarter, and a half-dollar. Each has a head and tail. The number of heads and tails which can appear when the coins, or any number of coins, are on the table below.

Let's record some data.

1. Use one coin. Notice we can have one head (H) or no head.
2. Use two coins. Notice we can now have two heads, ___ head or, no head.

3. Use three coins. Now we can have ____heads, ____heads, ____head, or ____head.
4. Use four coins. Record your findings below.
5. Try five coins.

Number of coins	Number of heads possible	Number of tails possible
1	1 0	0 1
2	2 ___ 0	0 ___ 2
3	___ ___ ___ ___	0 1 ___ 3
4	___ ___ ___ ___ ___	___ ___ ___ ___ ___
5	___ ___ ___ ___ ___ ___	___ ___ ___ ___ ___ ___

With the coins above we find it possible to record the data in another way.

2 coins	HH	HT TH	TT		
3 coins	HHH	HHT HTH THH	HTT THT TTH	TTT	
4 coins	HHHH	HHHT ___ ___ ___	HHTT ___ ___ ___ ___ ___	HTTT ___ ___ ___	TTTT

Other types of cues and prompts, depending for the most parrt on the ability of the writer, include:

1) visual cues, use of second color, italic or other typographical variations (capital letters, bold face, etc.), diagrams with accented lines

2) conceptual cues, similarities, contrasts, references to the whole topic, analogy

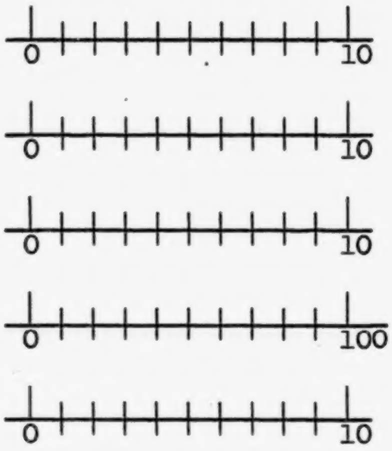
3) mechanical cues, verbal patterns, rote responses, word pairs, etc.

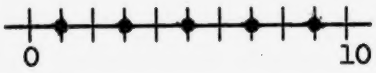
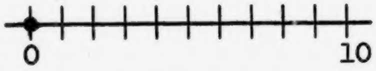
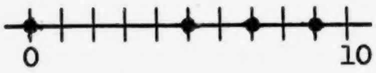
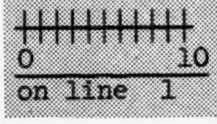
The term "panel" was introduced into the jargon of programed learning when referring to material which was apart (like an appendix) from the program itself, as, for example, tables of statistics, charts, pages of text, summaries, etc. One of the characteristics of the SMSG efforts has been to develop panels constructed in part by the student. The use of the constructed panel is varied. It can serve as a summary of properties, for example, which are studied throughout the chapter and recorded by the student as he completes each one. The panel may record and preserve the totality of a proof on which the student is working. The panel may summarize experimental data gathered to which the student may later be referred. The panel may preserve the equation as a first step, the table of values as a second step, and the graph as a third step. A panel may simply display information or illustrations which are too bulky to place in the text.

Let us consider a few examples of pupil-constructed panels:

PANEL I

1. $S = \{0, 5, 7, 9\}$
2. $T = \{0, 2, 4, 6, 8, 10\}$
3. ____________________
4. ____________________
5. $M = \{0, 1, 3, 5, \ldots\}$



	Items 17-25 refer to Panel I. Open to Panel I	
17	Line 1 reads "S = {0, 5, ☐, ☐}" (The underscore in the response column indicates that the material is to be copied onto the Panel in the space indicated.)	7, 9
18	Let K be the set of numbers that are elements of S and also elements of T. K = ☐	K = {0} on line 3
19	K ______ a subset of T. (is, is not)	is
20	Let U be the set consisting of the squares of the elements in S. Then U = { }.	U = {0,25,49,81} on line 4
21	Which of the following is the graph of set S? [A]  [B]  [C] 	 on line 1

Clearly [A] is the graph of the set {1, 3, 5, 7, 9}. Is this the set S? [B] is the graph of K, [C] is the graph of set S? If you were correct, try graphing sets T, K, U, and M.

Notice that the first reference to the panel makes sure that the student has the correct panel. If the panel is on the response sheet it may be used in the following manner:

Suppose a student is to complete a proof on his own. He could be told in the text that he should work on the Panel until he needs help, then refer to the item which gave him trouble. If a less able student does not work through the Panel alone, he may do so with guidance from the text. The following example illustrates the interplay between text and response sheet (on which the panel is located):

Try to complete the proof of this theorem for yourself. On your response sheet is an outline of a proof (see Panel 7-2). Do not refer to the items below unless you find blanks in the proof which you cannot complete. In such a case, refer to the item below which is numbered to correspond to the one which gave you some trouble.

THEOREM: If a is any real number, then a·1 = a and 1·a = a.

22 The theorem we are about to prove deals with the _____
property of 1. We have already shown that this property
23 holds for all __________ numbers.

multiplication
nonnegative

24 CASE I assumes then that a is a ________ number,
25 for which the __________property of 1 is known to
hold. Here, there is no problem; when a is nonnegative,
a·1 = a.

nonnegative
multiplication

26 Case II assumes that a is a _______ number. To
prove that this property still holds, we go back to the
27 __________ of multiplication for real numbers. This
definition states (in part) that if one of the numbers is
28 29 _________ and the other is __________, then

$$ab = -(|a|\cdot|b|).$$

We know that |1| = 1 by the definition of absolute value,
and we know that in this case a is negative, so

$$a\cdot 1 = -(|a|\cdot 1).$$

30 |a| is nonnegative so |a|·1 = ____. The opposite of
31 32 (|a|.1) is ____; so too the opposite of |a| is ______.
Since a is negative and |a| is then the opposite of
33 a, we can write |a| = ____. Further, the opposite of
the opposite of a real number is the number itself, so
34 -|a| is another name for -(-a) when _____ is negative.
We have thus shown that when a is negative, -|a| =
-(-a) = a.

negative
definition
nonnegative,
negative
|a|
-(|a|·1),
-|a|
-a
a

The proof is now complete because each real number is either nonnegative or negative; in both cases above, we have shown that $a \cdot 1 = a$. Check your completed proof with the one given on the next page. Note that there may be more than one proof for the theorem, but that the one given is sufficient to demonstrate the truth of the theorem.

PANEL 7-2

THEOREM: If a is any real number, then $a \cdot 1 = a$.

PROOF:

CASE I: Assume that a is nonnegative

Then, if a is nonnegative, $a \cdot 1 = a$ because the

22 ______________________ property of 1 is known to be true for all

23 ______________________ real numbers.

CASE II: Assume that a is negative.

26 Since in this case a is ____________ and 1 is

27 nonnegative, $a \cdot 1 = -(|a| \cdot |1|)$ by the _________ of multiplication of real numbers; since $|1| = 1$ by the definition of absolute value, we can write

$$\begin{aligned} a \cdot 1 &= -(|a| \cdot |1|) \\ &= -(|a| \cdot 1) \end{aligned}$$

and use the multiplication property of 1 for nonnegative real numbers to write

32 $$-(|a| \cdot 1) = -|a| \ .$$

33 But, since a is negative, $|a| =$ _________ by definition of

34 absolute value; hence, $-|a| = -(-a)$ or _____ because the opposite of the opposite of a real number is the number itself.

In both cases, we have shown that $a \cdot 1 = a$ is ________________ (true, false) when a is any real number.

Check your completed proof with the one given on page xx.

2-5 Revision: Comments and Criticisms

So you thought your first draft was good, eh? Well, a mathematician, a teacher of high school mathematics, a psychologist, and a high school student will all be willing to read your prose and offer suggestions. They may, as some have, say, "You forgot a comma in line 3, a semicolon before "therefore" in line 7 and misspelled "hypotenuse" on page two." The "fly-speckers" will find most of your typographical and grammatical errors on a first or second reading and save you from the critics in the published reviews later. Don't underrate them!

You may find your prose being suggested for the booby prize, since you "failed to specify the domain of the variable in the equations on page 2, and again on page 3 where failure to specify the domain allows the equation to become either indeterminate or meaningless, depending on the value chosen." The same critic may call your writing "mathematical nonsense". The critics should be willing to be brutally frank, and you must be prepared to hear their complaints. You should plan to do the same for others.

It is possible that the teacher-reviewer will claim that "if the wording were simplified, my ninth graders could understand it. Why not use a few more concrete cases like those on page ____ of the text?"

The psychologist may offer criticism when he understands the mathematics. He may suggest that "the student does not seem to be prepared for this particular notion." Other comments might be: "Does this response really add to understanding of the problem?" "Why repeat this question as often?" "Wouldn't it be better to schedule the item in later for review purposes?" "The shift to this new topic is abrupt, how about some transition material?"

"The topic is well-developed and logically presented but could use another illustrative example before going on." Why not say, "For all real numbers, a, b, and c, $(a + b) c = ac + bc$ or $c(a+b) = ca + cb$, since the 'or' is valid whether the system is commutative or not. Right and left-handedness should not concern the student at the elementary level." "Do you want the student to prove this or follow your proof?" "I would guess that at the rate of 60 items per hour, the program would last 250 hours. There are only 150 usable hours in a school year. Is this reasonable?" "Most students have trouble here. More drill is necessary." "Why belabor this?"

From the student, or an equally naive subject, you may find that "the wording was ambiguous. I thought of at least five ways to complete the sentence correctly." ($x^2 + 3x + 2$ is _____ .) Or, "Why did you include so many examples on the easy topics and so few on the difficult ones?"

2-6 Revision and the Second Draft

The critics of the first draft are clearly not supposed to be gentle. If they are, your criticism later may be brutal. What use should authors make of criticism?

In some cases, the comments require simply correction of typographical errors and minor changes. These are easily done by persons expert in English but not necessarily expert in mathematics. But beware--sometimes a minor change of wording may change the meaning considerably. Recall R.E.K. Rourke's experience on the Report of the Commission on Mathematics (CEEB) when he returned from Russia to find that the editor had found "real" too redundant so had changed some of the occurrences to "genuine" numbers. The change from "into" to "onto" could also be nontrivial.

A goodly number of the comments are likely to be very general and force the revision of several items and/or frames. Some will deal with clarity of presentation, some with length of sentences and paragraphs, and some with the mathematics per se.

One of the findings which bears notice here is that in attempting to write mathematics for a self-instruction text forces the author to think more critically than ever before. The programed text, which may or may not work, brings out the good and the bad in details of the mathematics and denies authors the glibness which they assume in conventional writing. In this important way, programing is as good for the authors as it is for the students. Authors of programed texts must be prepared for the possibility that the mathematics, when approached from a learner's point of view, may be different in structure from the mathematics when seen by the mathematician.

The following questions may be asked in the course of the writing. The answers will vary considerably but cognizance of their presence may preclude some of the common mistakes made by many programers.

1. What content is being taught? What kinds of student behavior are desired as end products? What student behavior exists at the beginning

of the program? In what way is the content best presented? Is programing the material an efficient expenditure of energy?

2. How long would a conventionally taught student spend on this material? Is the program to do the whole job of teaching? Are teachers films, or tapes available to the student? Does the program utilize non-text materials as resources?

3. Are the frames too long? Is a frame filled with too much detail? Are the responses relevant to the skill, vocabulary, or concept being developed? If a mistake is anticipated, are corrective tracks provided? Are items poorly worded so that students will err? Are mistakes capitalized on by programers?

4. Can the response be made without reading the entire frame? Is the response over-cued? Is the language appropriate to the student level? Is the wording artificial? Is the response repeated very often? Has a verbal pattern been fixed by the repeated use of an expression? Are the responses accumulating to the total behavior desired? Can the student see where he is going? Are there too many frames in the unit? Are panels used effectively? for summary? for exploration? Can the student move into and out of the panels effectively?

5. Important. Did the programer list the techniques which seem to work some of the time as a guide to future writing?

The second draft gives the programer a chance to work on one very important factor: style. The use of illustrative material and consistency within frames can be improved at this point. The programers should be able to read all antecedent sections and make transitions smooth.

Editing a programed text is not like editing a conventional text. The editor cannot scan, omit pages to be done later, change punctuation and/or wording, or rearrange passages randomly. In a program, the items and frames are written for the purpose of shaping behavior and, insofar as they accomplish this end, they can withstand little revision. (That is not to say there's only one way to shape behavior, but another path may be widely separate from the one chosen by the authors in a given program.) The editing of a programed text is necessarily almost continuous. Students, critics, and teachers may raise distinct questions.

In the early days of the "art", error rate was considered an important factor. The magic number (5 percent) seemed to prevail. Seldom did authors raise the more relevant issues: Are the mistakes trivial or significant? Does low error rate really say anything about the learning which is taking place?

2-7 Student Try-out: Small and Large Scale Evaluation

Barlow (of Emory University) once called programed instruction, "Validated instruction". By this he meant simply that the material was validated by actual student trial before being unleashed on a large population. This notion has also been widely propagated by other psychologists who have maintained that "now we will truly get instruction at the student's level." Beware!! The response elicited is in fact a sample of behavior and may not reflect knowledge of the whole. If a program is revised after one student try-out, we can expect another student to have made some different responses and reactions. With two or three students we begin to find trends in response patterns and, after taking a sample of ten or fifteen, either the data reduction becomes a burden or the response pattern has stabilized.

During the summer of 1963, a student was found to go through the first eight chapters of Form H. Problems involved: (1) he was above average ability; (2) he could work sections consecutively only if they were written and ready to read; (3) he was motivated by remuneration and the thought of skipping the course in his own school; (4) he could work as much as six hours in a day.

Some programers have failed to note that there may be a cumulative effect of a student going through a program and have reported different chapters checked on different groups of students (many who had not worked the previous chapters in programed form.) It may be less acceptable but more realistic to get experienced teachers to work on the materials and wait for a finished product to get evaluation.

Just as a single student provides learner data, a carefully designed experiment can produce group data. The variables may include:

1. grade level
2. varieties of presentation such as
    a. self-paced
    b. group-paced
    c. used as supplement to a text
    d. used as replacement for a text

3. variations of the program per se, such as

   a. size of step changes
   b. vocabulary changes
   c. response mode
   d. order of topics being presented.

Large populations can be found in public and private schools on which experimentation can be performed. Teachers vary from receptive to antagonistic and influence their students accordingly. The large scale experiment involves a problem of data reduction on a grand scale. To undertake it without adequate preparation is suicidal.

3-0 Program Analysis

When is a program a "good" program? A variety of answers have been proposed in the past few years and others remain open to further investigation. In the next few sections, presentation of some techniques for analysis of programs by those whose concern is the production of a "good" program will be attempted. Bear in mind that the criteria here are not designed to evaluate content, but form; not use, but construction.

3-1 Criterion Items

Early in the movement, psychologists and some programers maintained that it would be possible to isolate criterion frames or items and to measure the program in terms of achievement on those items. The questions raised by this technique hinge on two basic issues: The validity of the question itself and the role of the question in the shaping of final behavior. Clearly a question may fail to test what it purports to test or may fail to discriminate among students of widely diverse ability. Further, one can ask which items are criterion items along the path and fail to see the total behavior shaping up.

One of the characteristics of contemporary mathematics instruction is the spiral nature of the subject. The number line may occur in the very first chapter and be woven through the fabric of the course as a unifying thread. Should a "thorough understanding" (whatever that means) of the distributive property be required when the student is just beginning to study mathematics? Or should synthetic proof be forced into false bloom in geometry before an intuitive grasp of the notion of proof is acquired?

The fact that criterion frames are not yet shown to be reliable guides to the revision of programed material should not be taken to mean that criterion frames or items are not useful. In fact, teachers use criterion questions during the course of instruction to see if a student is grasping a given concept. The criterion item may demand recall, recognition, extrapolation, analysis, or any of the other behaviors which are sought en route to the terminal goals. Their value in programed instruction is not unlike their value in the classroom--they tend to shape the instructor's course of action.

3-2 Flow Charts and Concept Matrices

In the writing of a program, it may help to have a flow chart or matrix showing the accumulation of ideas. Flow charts may be similar to the task analysis charts of Gagné or simply a listing of topics with their antecedents explicitly noted.

A more sophisticated format was suggested by the IBM Guide to Writing Programs. It was known as the "concept matrix", a rectangular matrix in which rows contain the concepts and columns contain item numbers. Within the matrix a cross indicates a definition, a dot represents a relationship containing the concept, and "C" represents a contrast item. In one sense, the concept matrix represents a flow chart to show the introduction, definition, relation, and contrasts of newly developed ideas. Such a scheme, if outlined before items are written could be of assistance in fading schedules and determining critical points for the insertion of criterion items.

3-3 Logical Analysis

In the programing of the SMSG First Course in Algebra it became clear that some topics were easier to program than others. Which ones? But more important --Why? Recall that this effort was essentially a translation from existing text. The content and structure of the text were preserved for the most part. Let's look at the difficult spots.

1. An early section on the number line. Why? What is assumed? What should a student know when he has completed this section? What information is to be included? rational numbers? successors? betweenness?
2. The chapter on the multiplication of real numbers. Why? Is it parallel to addition? What cases can be proved? How much proof is appropriate? How long should the material take?

3. Manipulation with radicals. Why? What earlier material is needed? what exceptions should be pointed out? Are proofs necessary? What should the student be able to do when he has studied radicals?

4. Polynomials. Why? What are they? Are we to consider their form apart from their roles as functions? How shall we develop factorization?

These are some of the trouble spots. What appears as we ask where trouble starts is a conjecture: Topics for which the subject matter does not force an explicit orderly development are hardest to program, to write, to teach, or to learn. In spite of the often quoted "Mathematics is a logical science," there appear to be aspects about which we know little at this time--among them, how students learn mathematics. Perhaps when we know more, we shall be able to shed the shackles and teach.

3-4 Data: Collection and Reduction

To guide teachers as users and programers as generators of materials, certain data have been shown to be effective. To those undertaking the programing of mathematics, the task may seem monumental, but it bears good fruit.

A series of tests were constructed to measure achievement in algebra. These included two pre-tests to see what background was on hand, and a series of chapter tests, and two post-tests to measure final achievement. A standardized aptitude test was given to enable grouping into quartiles for analysis purposes. A sample of 100 students were randomly selected from the population and their tests analyzed. The following data has been useful to the programers:

1. On each item on a chapter test
    a. the percent of students scoring correctly on the item;
    b. the percent of students missing the item;
    c. the correlation of each item with the total score on the test;
    d. the number of students in each SCAT quartile who missed each item;
    e. a histogram showing the error frequency for each item;
2. On the total test
    a. mean score;
    b. standard deviation;
    c. a histogram showing total score frequency;

3. On the ability test
    a. mean score;
    b. standard deviation;
    c. a histogram showing total score frequency;
4. Teacher reactions
5. Student opinions

In the end, analysis of pre and post testing can be made as well as correlations of: ability measure with performance, specific items with pre and post testing, and other factors (such as reading) with specific sections or skills (such as problem solving).

To facilitate data collection and reduction, forms can be devised for use by teachers and/or students. Tailor-made response sheets have been found to be more advantageous than a fixed response sheet format. Teachers, when loaded with a myriad of other teaching duties, are "reluctant" (at best) to engage in data accumulation. Students may be provided identification numbers at the start, classes or schools may be identified by number, rosters can be dittoed, students should be relied on to provide accurate data on their time and/or errors.

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APPENDIX 1 - ADDENDA BASED ON 1964 EXPERIENCE

A1-1 The Relation between Program and Text

The original version of the programed *First Course in Algebra* followed closely the SMSG text *First Course in Algebra* in that the order of presentation, the examples, and the exercises were those of the text. As noted earlier, careful section by section analysis of content, behavioral goals, and teaching sequence is the first phase of programing. Thus the availability of a well-designed text reduces the time required for this phase. However, it is an over-simplification to regard the ideal program--were one to exist--as simply a more elaborate version of a corresponding ideal text.

A text is designed for use in a class, and the programer at certain points is programing the behavior of a teacher as much as the text itself. For example, a text may include exercises which lead into the next topic to be covered. The purpose of such exercises is to motivate, and this purpose may be served even though few students are able to solve the problems without help. Simply to program the solutions completely would diminish rather than enhance interest in what follows.

Again, a text may use a rather complicated example, illustrating several ideas. The teacher can use such an example as the basis for class discussion, perhaps after students have read the material in the text. However, programing a lengthy example before background has been developed may create a bulky and difficult passage in which the main ideas are lost.

A teacher is often expected to supply ideas or interpretations not explicitly covered in a text. To help the teacher serve as a resource in this way, a commentary often supplies the teacher with information not given in detail in the text. Presumably, the teacher's comments on a question or discussion relate the student's ideas to more advanced or extended concepts. For the programer, the commentary presents a special sort of problem. It is desirable to convey to the student a sense of wider horizons without elaborating as fully as does the commentary. Thus occasionally, the program includes phrases such as "You will find later that...", without requiring any explicit response from the student.

In a spiral curriculum, it is not expected that an idea will be mastered on the student's first encounter with it. Similarly, it is appropriate to present material which the student is expected to read but not necessarily to master fully. For example, in forming an idea of Proof, some students profit from seeing proofs before they are able to construct them independently. The use of panels with constructed response items referring directly to them, is a way of engaging attention in such instances.

If a text contains lengthy problem lists, teachers may assign only selected problems from them. The programer, on the other hand, must make a decision about the number of problems to be included.

While observation or tests show that students have not mastered certain material well, teacher-critics tend to ask for more problems and drill. Sometimes, however, it appears probable that a different organization and presentation would be more helpful than additional drill. For example, the material on factoring polynomials caused great difficulty even though a first revision had incorporated more examples than had the original draft. It appeared possible that this topic is one where the teacher's writing on the blackboard is important by providing a visual, moving stimulus. Thus, it is plausible that rearranging the presentation rather than adding problems is called for here.

Even experienced teachers find that the experience of writing a program can be rewarding and provocative, though it is arduous. It calls for a fresh view of the elements involved in teaching. Likewise, it has been suggested that programed materials might be of value to inexperienced teachers in planning presentations. For the new teacher, the program, which is far more detailed than a text, may serve as a helpful guide to systematic organization and to the establishment of objectives.

A1-2 Special Features of a "Hybrid" Program

As illustrated earlier, the later versions of the program, Forms H, contain multiple-choice and constructed response items as well as passages of straight text. Different teaching situations lend themselves more readily to one mode of instruction or another, and the programer using a "hybrid" form tends to develop a feeling that for any given purpose there is usually a mode that is particularly apt. Use of multiple-choice questions appears natural, for example, when one wishes to warn the student against a certain error, to discriminate between fine shades of meaning, or to remind the student of the reason that a certain response is incorrect. A multiple-choice question may also be used, as on page 57, in situations where several answers are appropriate, or in general, where comment on alternative answers is called for. A constructed response form, on the other hand, often appears preferable for the step by step development of an idea. The programer sometimes varies the type of item simply to avoid monotony.

Teachers have commented that when students use programs exclusively, they may fail to develop habits of organizing written work in an orderly way. For this reason, the complete solutions to certain lists of problems are given in the answer key at the back of the book. Students are not told that their work must duplicate the form of the solution given, but it is hoped that by comparing their work with acceptable models, they will become more proficient at presenting complete solutions.

Almost every chapter ends with a summary which is not programed. The summary (See APPENDIX 2) is intended to make the book useful for review as well as to encourage the student to organize the main ideas that have been included in a chapter. Aside from the use of nonprogramed material to summarize developments that have been presented and to offer relief from strict programing, expository sections are also used to introduce chapters by placing the material within specific frames of reference.

The inclusion of nonprogramed material is expected to make the program more adaptable to situations where its use is supplemented by teacher-directed classroom activities. It appears that programs will be used in this way in some schools. For this reason, the teachers' commentary suggests to teachers appropriate techniques for the purpose.

Some critics have called attention to the danger that the techniques of programing create the idea that each problem can be solved in only one way. To avoid such rigidity, alternative methods of solving a problem are often included. Multiple-choice questions with more than one acceptable answer are sometimes used, as in the example on page 57. In some cases, students are directed to try to solve a problem for themselves and refer to the detailed solution given in the program only if they have difficulty.

To provide for individual differences, various procedures may be used. For example, a theorem may be stated on the basis of several examples and the formal proof made optional. As an illustration, the following may be cited:

Items 37-41 suggest

Theorem 12-4c. For positive integers a, b, and c, if a is factor of b, and a is a factor of (b + c), then a is a factor of c.

*42. If you wish, try to prove Theorem 12-4c and then turn to page i to see one method of proving it.

Problem lists may also include optional, more difficult examples for more able students. Moreover, some relatively simple problems lists for drill are incorporated in a way that permits skipping by more able students. Typical accompanying instructions are: "If you had difficulty with these Items, do Item ____ to Item ____ for further practice," and "If you answered Item ____ correctly, proceed to Item ____." This type of branching may be used for either constructed response or multiple-choice items.

APPENDIX 2 - SAMPLE CHAPTER SUMMARY*

CHAPTER 7 - Properties of Addition

We have defined addition of real numbers as follows:

The sum of two nonnegative numbers is familiar from arithmetic.

The sum of two negative numbers is negative; the absolute value of this sum is the sum of the absolute values of the numbers.

The sum of numbers, of which one is positive (or 0) and the other is negative, is obtained as follows:

The absolute value of the sum is the difference of the absolute values of the numbers.

The sum is positive if the positive numbers has the greater absolute value.

The sum is negative if the negative number has the greater absolute value.

The sum is 0 if the positive and negative numbers have the same absolute value.

We have satisfied ourselves that the following properties hold for addition of real numbers:

Commutative Property of Addition: For any two real numbers a and b,
$a + b = b + a$.

Associative Property of Addition: For any real numbers a, b, and c,
$(a + b) + c = a + (b + c)$.

Addition Property of Opposites: For every real number a,
$a + (-a) = 0$ and $(-a) + a = 0$.

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* This is an example of chapter summaries in the student text (See page 84 of Manual) and is not illustrative of the chapter summaries and objectives prepared by Haag, Rickart, and Pollak (page 3) as a guide in the writing.

Addition Property of 0: For every real number a,

$$a + 0 = a \text{ and } 0 + a = a.$$

We also stated a fact, already clear from our earlier ideas, which we called the *addition property of equality*:

For any real numbers a, b, and c,

$$\text{if } a = b, \text{ then } a + c = b + c \text{ and } c + a = c + b.$$

This idea supplied us with a useful procedure for determining the truth sets of open sentences.

We have proved that the additive inverse is unique--that is, that each number has exactly one additive inverse, which we call its opposite.

We have discovered and proved the fact that the opposite of the sum of two numbers is the same as the sum of their opposites.

APPENDIX 3 - SAMPLE SUMMARY OF CHAPTERS

10-5 The Real Numbers - Summary

Let us summarize the material of Chapters 6-10.

We began by introducing the set of negative numbers, as suggested by our knowledge of the numbers of arithmetic and the number line. We called the union of the set of negative numbers and the set of numbers of arithmetic the set of real numbers.

Addition and multiplication of real numbers were then defined in such a way that

1. the usual meanings of addition and multiplication in the set of numbers of arithmetic were preserved, and
2. many of the properties of these operations, which were true for the set of numbers of arithmetic, were also true in the set of real numbers.

We discussed an order relation in the set of real numbers, again based on our ideas of order in the set of numbers of arithmetic.

In the course of this development we stated many properties of addition, multiplication, and order.

Here is a list of certain basic properties which we considered:

For any real numbers a, b, and c,

1. $a + b$ is a unique real number.

1'. ab is a unique real number.

2. $a + b = b + a$

2'. $ab = ba$

3. $(a + b) + c = a + (b + c)$

3'. $(ab)c = a(bc)$

4. There is a special real number 0 such that $a + 0 = a$ and $0 + a = a$.

4'. There is a special real number 1 such that $a \cdot 1 = a$ and $1 \cdot a = a$.

5. For each a there is a unique real number $(-a)$ such that $a + (-a) = 0$ and $(-a) + a = 0$

5'. For each a, different from 0, there is a unique real number $(\frac{1}{a})$ such that

$$a \cdot \frac{1}{a} = 1 \text{ and } \frac{1}{a} = 1.$$

6. $a(b + c) = ab + ac$, $(b + c)a = ba + ca$.

7. For a and b, exactly one of the following is true:

$$a < b, \quad a = b, \quad b < a .$$

8. If $a < b$ and $b < c$, then $a < c$.

9. If $a < b$, then $a + c < b + c$.

These are by no means all of the properties we discovered, but we will interrupt our listing at this point to make the following remark. If we add just one more basic property, we have a list of basic properties that could be used to prove everything about the real numbers. Unfortunately, this additional property would involve us in mathematics beyond the scope of this course.

Practically all the algebra of this course may be developed from the fourteen properties listed above. Let us now list some of the more important properties which may be proved as consequences of the basic properties. All of these have been discussed in the text. As you study more mathematics you will learn much more about the real numbers.

1. For real numbers a, b, and c, if $a + c = b + c$, then $a = b$.
2. For real numbers a, b, and c with $c \neq 0$, if $ac = bc$, then $a = b$.
3. For real numbers a and b, $ab = 0$ if and only if $a = 0$ or $b = 0$.
4. For any real number a, $(-1)a = -a$.
5. For any real numbers a and b, $-(a + b) = (-a) + (-b)$.
6. For any real numbers a and b, $(-a)b = -(ab)$ and $(-a)(-b) = ab$.
7. The opposite of the opposite of a real number a is a.
8. The reciprocal of the reciprocal of a non-zero real number a is a.
9. For any non-zero real numbers a and b, $\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab}$.
10. If a and b are real numbers, then $a < b$, if and only if there is a positive number c such that $b = a + c$.

11. For any real numbers a, b, and c,

$$\text{if } a < b \text{ and } 0 < c, \text{ then}$$
$$ac < bc,$$
$$\text{if } a < b \text{ and } c < 0, \text{ then}$$
$$bc < ac.$$

12. For any real numbers a and b, if $a < b$, then $-b < -a$.

13. If $0 < a < b$, then $\frac{1}{b} < \frac{1}{a}$.

14. If $x \neq 0$, then x^2 is positive.

15. If $0 < a < b$, then $a^2 < b^2$.

If we consider the set of real numbers together with the operations "+" and "·", the relation "<", and the basic properties we may call this the real number system.

APPENDIX 4 - SAMPLE OBJECTIVES

...More detailed objectives of the course are given in the Preface to the Commentary for Teachers accompanying the First Course in Algebra. Any attempt to list objectives, however, tends to generalities. As an alternative, we list a set of sample questions each of which illustrates a particular objective of the course; in a parallel column we explain what the question is designed to test, or not test. There are three catagories of questions: language, structure, technique. It should be understood that these are sample questions and that no effort was made to include the type of routine questions that are needed in a good test. Many such routine questions can be found in the Suggested Test Items in the Commentary.

Suggested Test Items in the Commentary

Sample Questions for SMSG First Course in Algebra

Language

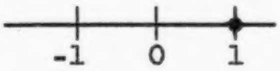
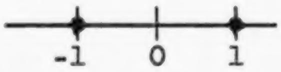
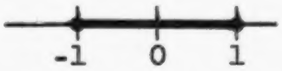
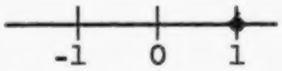
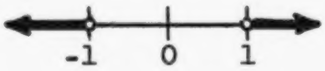
Item	Remarks
1. Another name for $2 + 3 \cdot 4 + 1$ is (a) $(2 + 3) \cdot 4 + 1$ (b) $(2 + 3) \cdot (4 + 1)$ (c) $2 + (3 \cdot 4) + 1$ (d) $2 + 3 \cdot (4 + 1)$ (e) $2 + (3 \cdot 4) + (3 \cdot 1)$	There are certain accepted conventions concerning preferred orders of operations.

Items	Remarks
4. Which of the following pairs of sentences have the same truth sets? I. $3(t - 4) + 5 = 4(t + 1)$; $3(x - 4) + 5 = 4(x + 1)$ II. $2x + y = 1$ and $y = 2x + 1$; $t = 0$ and $S = 1$ III. $\frac{x(x + 4)}{x} = 2$; $x + 4 = 2$ (a) only I (b) only II (c) only III (d) I and II (e) II and III	The equivalence of two sentences does not depend on the symbols used for the variables, but on their domains and the truth sets of the sentences. Here the domains must be implied from the sentences.
12. If f is defined by $f(x) = x + 1$ for all real x, then $f(f(t))$ is (a) $(t + 1)^2$ (b) $x + t + 1$ (c) $t(t + 1)$ (d) $2t + 1$ (e) $t + 2$	Language of functions involves the notation and its use.

Structure

Item	Remarks
22. Consider the set T of all numbers of the form $a + b\sqrt{2}$, where a, b, are any rational numbers. Which of the following is not true? (a) the set T is closed under addition (b) the set T is closed under multiplication (c) $0 + 0\sqrt{2}$ is the identity for addition in T (d) every element in T has an additive inverse (e) $1 + 1\sqrt{2}$ is the identity for multiplication in T	This is a difficult question, but it searches out the ideas involved in the structure of a field.

Technique

Item	Remarks
34. Which of the following is the graph of $x^2 - 1 > 0$? (a)  (b)  (c)  (d)  (e) 	Skill in solving and graphing inequalities.